

Texas Success Initiative (TSI)

MATH REVIEW

Algebra Review

ALGEBRA & FUNCTIONS

Variables and Algebraic Expressions

- The sum of a number and 5 means:

$$m + 5 \text{ or } 5 + m$$

- The number diminished or subtracted by 7 means:

$$m - 7$$

- Ten times a number means:

$$10m$$

- z divided by 3 means:

$$z / 3$$

Algebraic Expressions

- Key words which denotes addition
 - sum, plus, greater than, more than, larger than, rise, increase, gain
- Key words which denotes subtraction
 - difference, minus, less than, smaller than, fewer than, decrease, drop, reduce, diminish, lose
- Key words which denotes multiplication
 - multiplied by, times, product, twice
- Key words which denotes division
 - quotient, divided by, ratio, half

Solving Equations

Examples:

- If $3t - 7 = 5t$, then $6t = ?$

Solve for t ; then find $6t$

$$3t - 5t = 7$$

$$-2t = 7$$

$$t = -(7 / 2)$$

$$\text{Thus, } 6t = 6(-7/2) = \mathbf{-21}$$

Solving Equations

Examples:

- If $(x-1)/x = 20$, then $x = ?$

$$x-1 = 20x$$

$$x-20x = 1$$

$$-19x = 1;$$

$$\text{thus } \mathbf{x = -1/19}$$

Ratios & Proportions

- Ratio is a way of comparing two or more variables.
- It is written as $a:b$ or a/b
- Proportions are written as two ratios in terms of fractions equal to each other.

Ex Solve this proportion for s :

p is to q as s is to t

$$p/q = s/t$$

$$s = t(p/q)$$

Solving Equations

Examples:

- Variables x and y are directly proportional, and $y=2$ when $x=3$. What is the value of y when $x=9$?

$$y \propto x \rightarrow y = kx \text{ where } k \text{ is a constant}$$

$$2 = k(3) ; k = 2/3 \rightarrow y = (2/3)x$$

Now, we can substitute $x=9$ and find y :

$$y = (2/3) 9 = 6$$

Solving Equations

Word Problem:

- A group of people ordered soup and sandwiches for lunch. Each person in the group had either one soup or one sandwich. The sandwiches cost \$7.75 each and the soups cost \$4.50 each. If the total cost of all 18 lunches was \$113.50, how many sandwiches were ordered?

$$A + B = 18 \rightarrow A = 18 - B$$

$$4.50 A + 7.75 B = 113.50$$

$$4.50 (18 - B) + 7.75 B = 113.50$$

$$3.25 B = 32.5$$

$$B = 10$$

Solving for Two Unknown Systems of Equations

- Solve for x and y:

$$4x + 4y = 12 \rightarrow \text{Eq. 1}$$

$$\underline{2x + 3y = 7} \rightarrow \text{Eq. 2}$$

$$(\text{Eq. 2}) \times 2: \quad 4x + 6y = 14 \rightarrow \text{Eq. 3}$$

$$(\text{Eq. 3} - \text{Eq. 1}): \quad 2y = 2$$

$$y = 1$$

$$\text{Substitute } y=1 \text{ into Eq. 2:} \quad 2x + 3(1) = 7$$

$$2x = 4$$

$$x = 2$$

Monomials

Algebraic expression that consists of only one term. (Ex: $9y$, X^{12} , $2qst^2$)

Adding & Subtracting Monomials

- Follow the same rules as with signed numbers, provided that the terms are alike. You should add and subtract the coefficients only and leave the variables the same.

$$10 \mathbf{x^3yz} - 12 \mathbf{x^3yz} = -2 \mathbf{x^3yz}$$

Monomials

Multiplying Monomials

- Add the exponents of the same bases

Example: $(x^2)(x^3) = x^5$

- When monomials are raised to the power, multiply the exponents of each part of the monomial by the power to which it is being raised.

Example: $(3x^2y^3)^3 = 27x^6y^9$

Monomials

Dividing Monomials

- Subtract the exponent of the divisor (denominator) from the exponent of the dividend (numerator) of the same base.

Examples:

$$(x^{12}) / (x^2) = x^{10}$$

$$(x^6y^3) / (x^2y) = x^4y^2$$

Polynomials

Algebraic expression that consists of two or more terms. (Ex: $x^2 + y^2 + 9y$)

Adding & Subtracting Polynomials

- Add or subtract like terms. You should add and subtract the coefficients of like terms and leave the variables the same.

$$10 \mathbf{x^3yz} + 2\mathbf{xy} - 12 \mathbf{x^3yz} + 3\mathbf{xy} = -2 \mathbf{x^3yz} + 5\mathbf{xy}$$

Polynomials

Multiplying Polynomials

- Multiply each term in one polynomial by each term in the other polynomial. Then simplify if needed.

$$\text{Example: } (p+q)(p+q+r) = p^2 + pr + 2pq + q^2 + qr$$

Dividing Polynomials by Monomials

- Divide each term in the polynomial by the monomial.

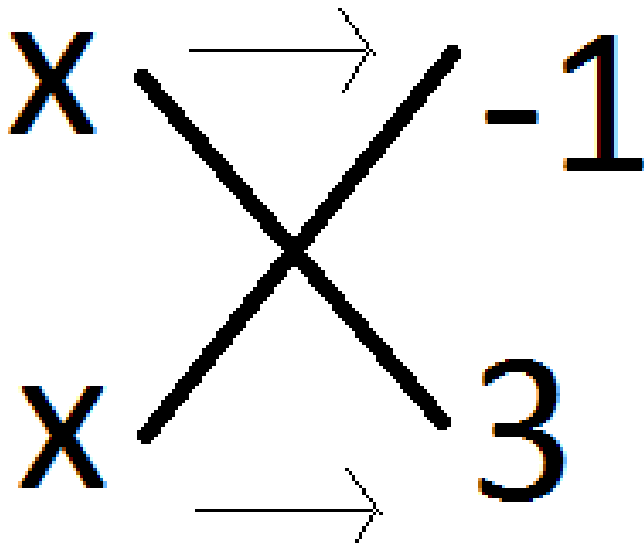
$$\text{Example: } (6y^2 + 2y) / (2y) = 3y + 1$$

Polynomials

Solving quadratic equations

Examples:

Factor $x^2 + 2x - 3 = 0$



$$(x-1)(x+3)=0$$

$$x=1 \text{ or } x=-3$$

Solving Equations

Examples:

- $x^4 - 1 = ?$

$$x^4 - 1 = (x^2 - 1)(x^2 + 1)$$

$$= (x - 1)(x + 1)(x^2 + 1)$$

$$x^2 - y^2 = (x - y)(x + y)$$

- If $\sqrt{5 - x} = 4$, then $x = ?$

$$5 - x = 4^2$$

$$-x = 11$$

$$x = -11$$

Linear Equation

- Equation of a straight line can be defined by the equation:

$$y = mx + c ;$$

where m is the slope of the line and c is the y -intercept in the xy -plane.

Linear Equation

Examples:

- Point C has coordinates (6,9).

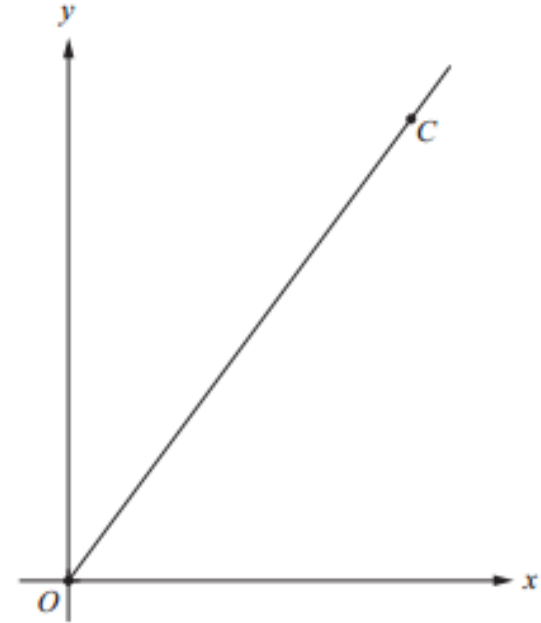
What is the equation of the line that contains points O and C?

$$y=mx+c;$$

$$\text{slope, } m = (y_2 - y_1) / (x_2 - x_1) = (9 - 0) / (6 - 0) = 3/2$$

$$\text{y-intercept, } c = 0$$

$$\text{Thus, } y = 3/2 x$$



X, Y Intercepts

Examples:

In the xy -plane, what is the y -intercept of the graph of the equation $y=2(x+3)(x-4)$?

y -intercept means $x=0$

Thus, substitute $x=0$ in the equation to find the y -intercept;

$$y=2(0+3)(0-4)$$

$$y = -24$$

Solving Inequalities

Treat them exactly like equations (except, if you multiply or divide both sides by a negative number, you must reverse the direction of the inequality).

$$\text{Ex: } -3x + 5 \leq 11$$

$$-3x \leq 11 - 5$$

$$-3x \leq 6$$

$$x \geq -2$$

Geometry and Measurement

Plane Geometry

- Shapes that can be drawn on a paper

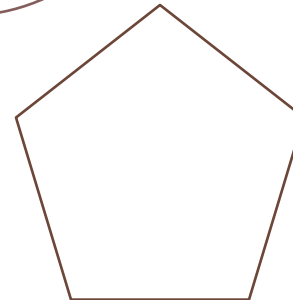
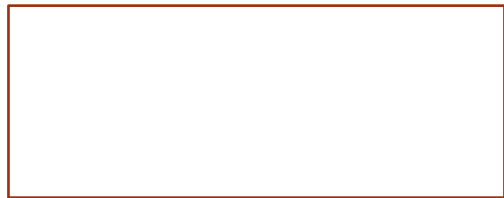
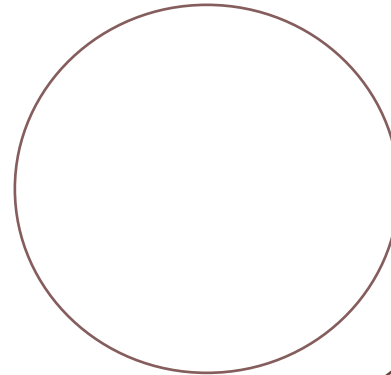
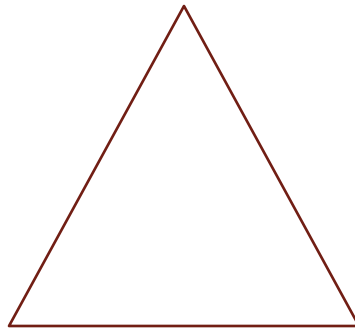
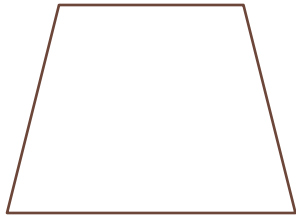
Point



Ray

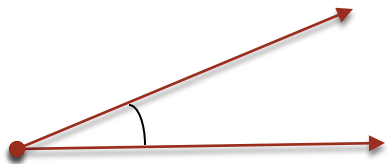
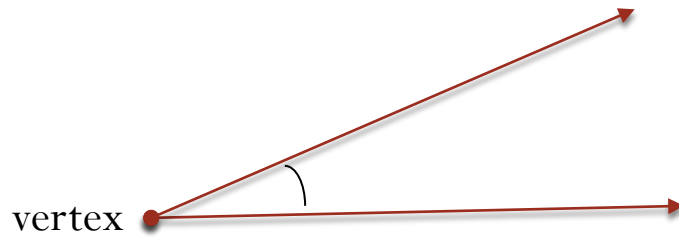


Line

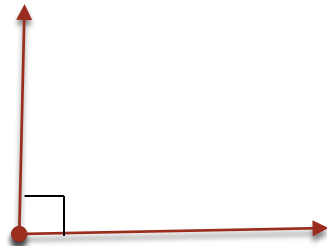


Angles

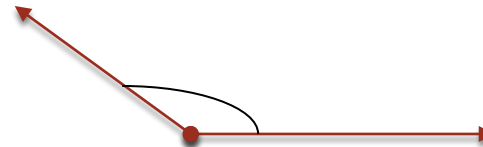
- **Angle** is formed by two rays that have the same endpoint called a vertex



Acute angle
measures less than 90°



Right angle
measures exactly 90°



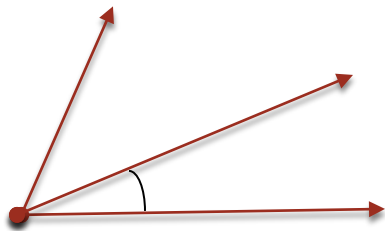
Obtuse angle
measures more than 90°



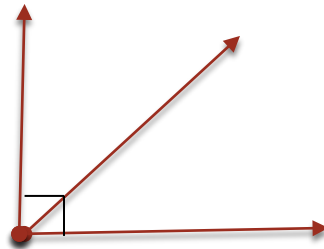
Straight angle
measures exactly 180°

Pairs of Angles

- **Adjacent Angles** are angles that share a common vertex and side
- **Complementary Angles** are adjacent angles together measure 90°
- **Supplementary Angles** are adjacent angles that together measure 180°



Adjacent angles

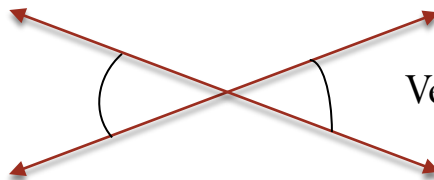


Complementary Angles



Supplementary Angles

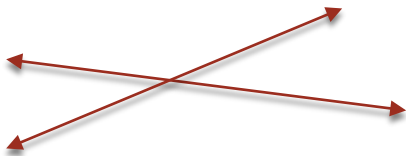
- **Vertical Angles** are opposite angles that share a common vertex



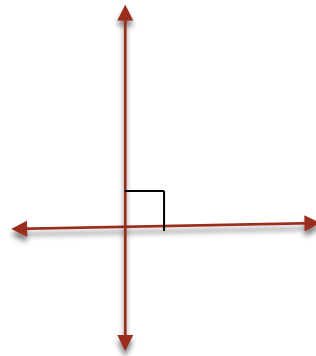
Vertical Angles are equal to each other

Types of Lines

- **Intersecting Lines** are two or more lines that meet at a point
- **Perpendicular Lines** Two lines that meet to form right angles
- **Parallel Lines** are two or more lines that remain the same distance apart. They never meet



Intersecting Lines



Perpendicular Lines

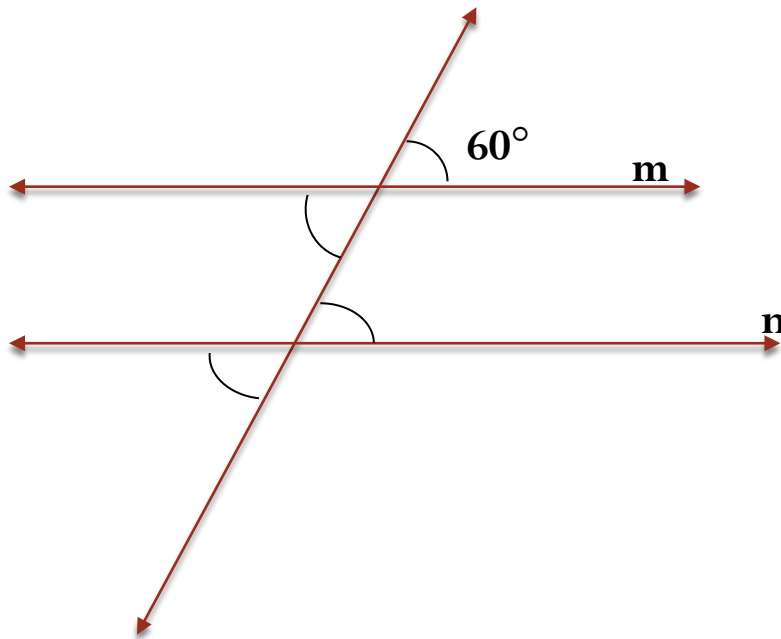


Parallel Lines

Angles and Lines

- Example

line m and line n are parallel. Fill in the missing adjacent angles.

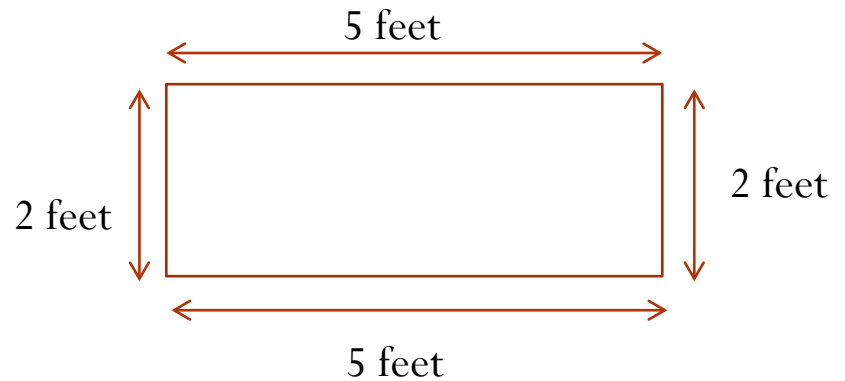


Measurements

PERIMETER

Total distance around the shape

$$5 + 5 + 2 + 2 = 14 \text{ feet}$$



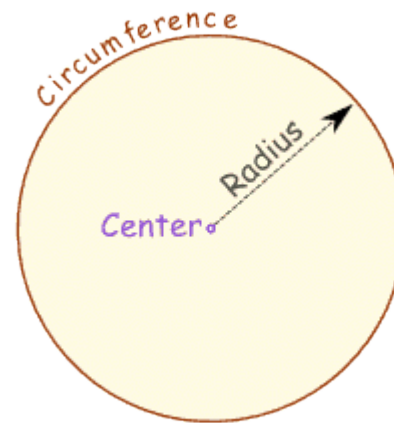
CIRCUMFERENCE

Distance around a circle

$$C = 2\pi r \text{ or } \pi d$$

Where r = radius d = diameter

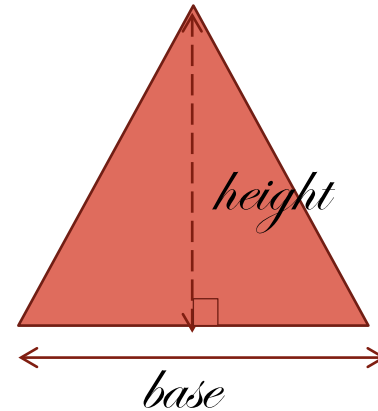
And $\pi \sim 3.14$



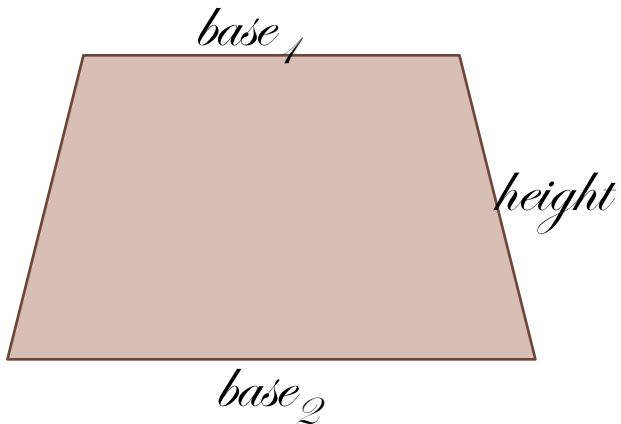
AREA



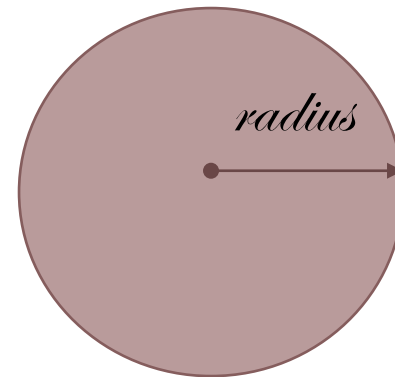
$$\text{Area of rectangle} = \text{length} \times \text{width}$$



$$\text{Area of triangle} = \frac{1}{2} \text{ base} \times \text{height}$$



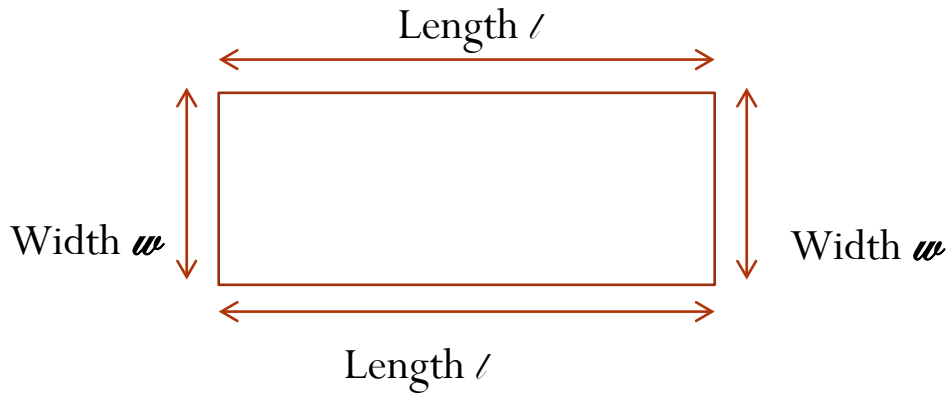
$$\text{Area of trapezoid} = \frac{1}{2} (\text{base}_1 + \text{base}_2) \times \text{height}$$



$$\text{Area of circle} = \Pi \times \text{radius}^2$$
$$\Pi \sim 3.14$$

AREA example

The yard behind Cindy's house is rectangular in shape and has a perimeter of 72 feet. If the length l of the yard is 18 feet longer than the width w of the yard, what is the area of the yard in square feet?



Perimeter of a rectangle

$$\text{Perimeter} = 2l + 2w$$

$$\text{Perimeter} = 72 \text{ feet}$$

$$72 = 2l + 2w$$

$$\text{Length } l = \text{width } w + 18$$

$$72 = 2(w + 18) + 2w$$

$$72 = 2w + 36 + 2w$$

$$4w = 36,$$

$$w = 9 \text{ feet}$$

$$l = 9 + 18 = 27 \text{ feet}$$

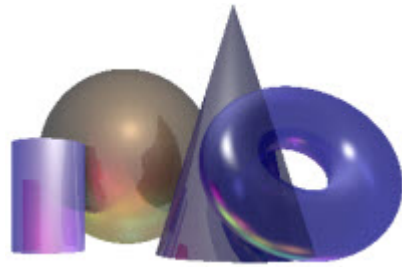
Area of a rectangle

$$\text{Area} = lw$$

$$\text{Area} = 27 \text{ feet} \times 9 \text{ feet}$$

$$= 27 \text{ feet} \times 9 \text{ feet}$$

$$= 243 \text{ square feet}$$



Solid or 3D Geometry

- 3 dimensional (width, depth, and height)

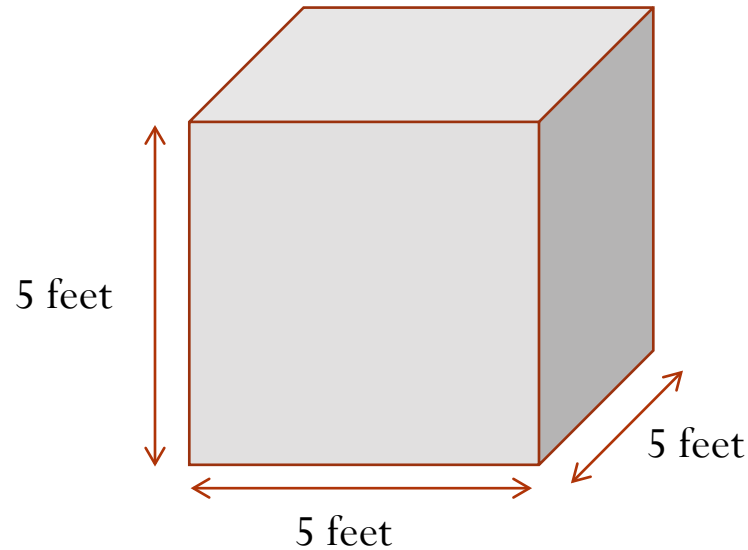


3D Measurements

SURFACE AREA

*Total **area** of all the surfaces
of a solid*

Surface area of a cube
= 5 feet X 5 feet X 6 surfaces
= 150 feet squared



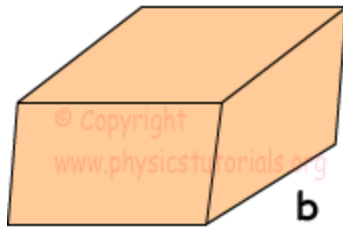
VOLUME

The total amount of 3D space a solid occupies

Volume of a cube = 5 feet X 5 feet X 5 feet
= 125 feet cubed

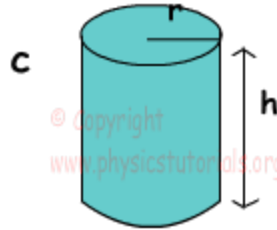
3D Measurements

rectangular prism



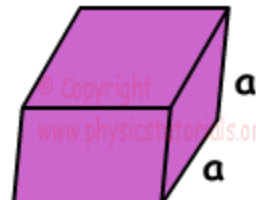
$$V = a \cdot b \cdot c$$

cylinder



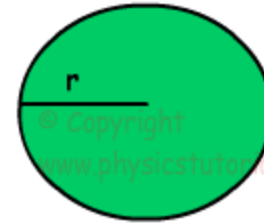
$$V = \pi r^2 h$$

cube

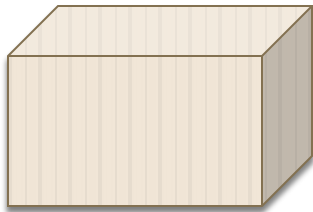


$$V = a \cdot a \cdot a = a^3$$

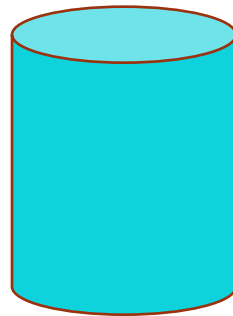
sphere



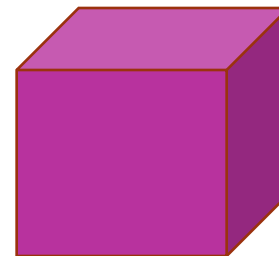
$$V = \frac{4}{3} \pi r^3$$



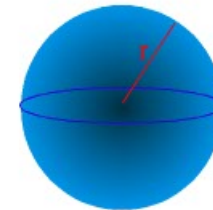
$$SA = 2ab + 2bc + 2ac$$



$$SA = 2\pi r^2 + 2\pi rh$$



$$SA = 6a^2$$



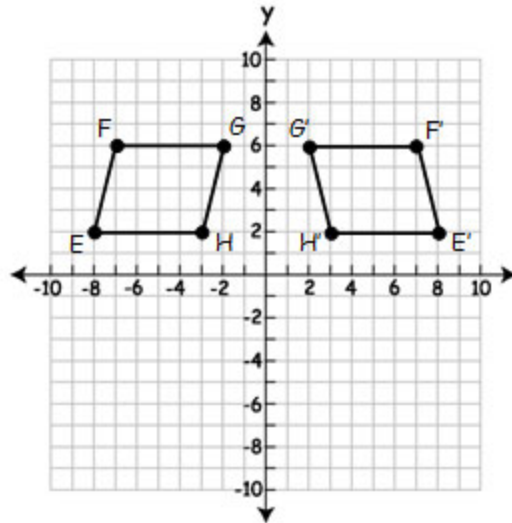
$$SA = 4\pi r^2$$

Transformations

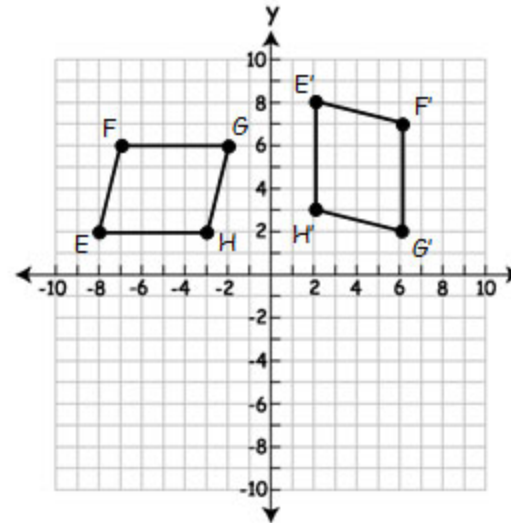
- *Rotation* means moving a shape around a point or line
- *Reflection* means reflected shape is same distance from a mirror line
- *Translation* means every point in shape must move in the same direction and distance

Which of the following figures has been rotated 90° clockwise about the origin?

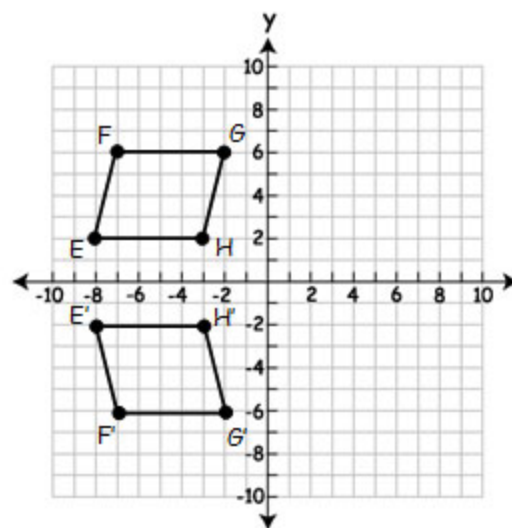
a.



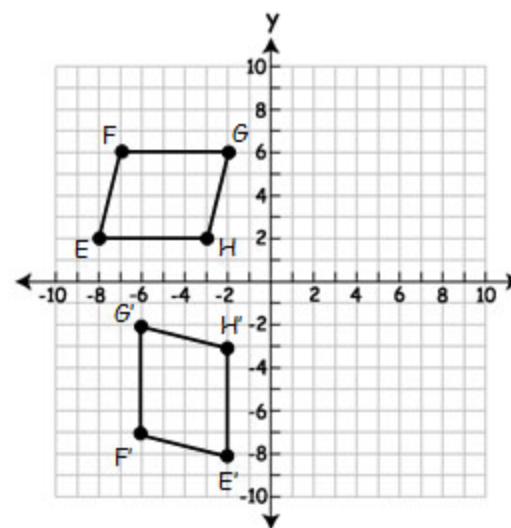
c.



b.

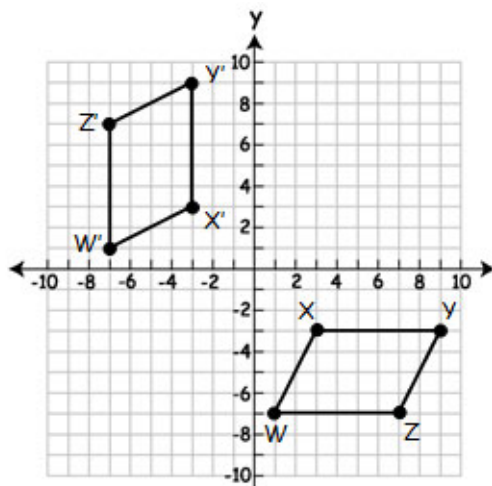


d.

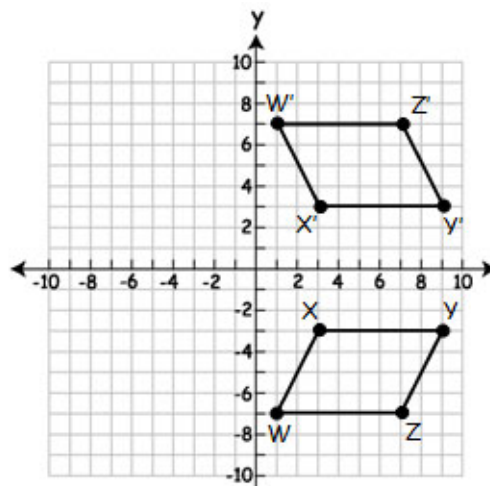


Which of the following figures show parallelogram $WXYZ$ being carried onto its image $W'X'Y'Z'$ by a reflection across the x -axis?

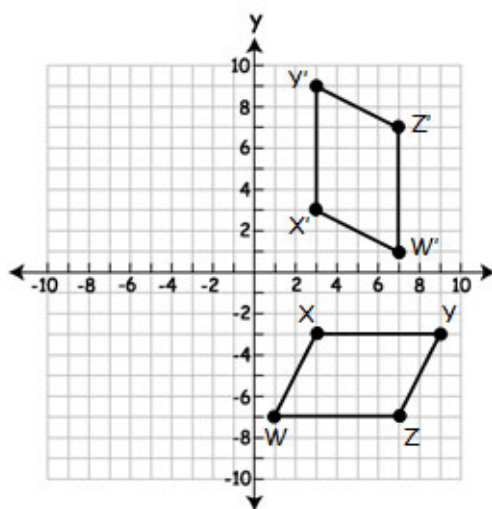
a.



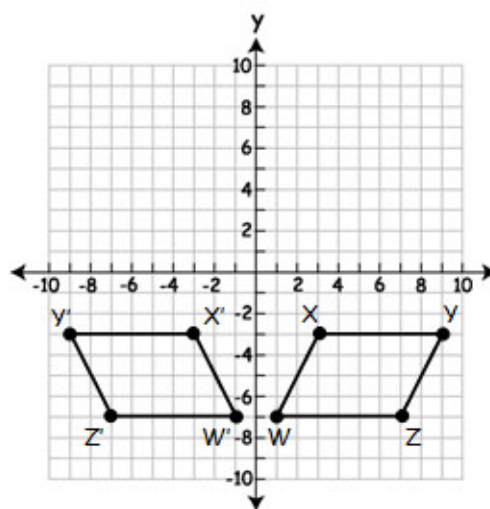
c.



b.



d.



Data Analysis, Statistics and Probability

Statistical Measures

- **Mean (Average)** = sum of group of numbers divided by the number of them

$$\frac{2 + 2 + 6 + 10 + 15}{5} = 35 \div 5 = 7$$

- **Median** is the middle number in a group of numbers listed in *ascending* or *descending* order

$$2, 2, \mathbf{6}, 10, 15$$

if there is an even number of items in the group, the median is the average of the middle two numbers

$$2, 2, \mathbf{6}, \mathbf{10}, 15, 20 \quad 6 + 10 = 16 \quad 16/2 = 8$$

- **Mode** is the number(s) that occurs most frequently

$$\mathbf{2}, \mathbf{2}, 6, 10, 15$$

- **Range** is the spread of the data. Range is found by taking the difference of the maximum and minimum values in the group.

$$2, 2, 6, 10, 15$$

$$\text{maximum} = 15 \text{ and minimum} = 2$$

$$\text{range} = 15 - 2 = 13$$

Statistical Measures

- Example

The table below shows the high temperature last Thursday for five cities, *A* through *E*. If the **median** of the Thursday high temperatures for these cities was 81°F , which of the following could **NOT** have been the high temperature last Thursday for City *A*?

City	High Temperature
A	$t^{\circ}\text{F}$
B	87°F
C	81°F
D	62°F
E	93°F



City	High Temperature
A	93°F
B	87°F
C	81°F
D	Less than 81°F
E	Less than 81°F

List the temps in ascending order starting with median

A. **85 too**

high

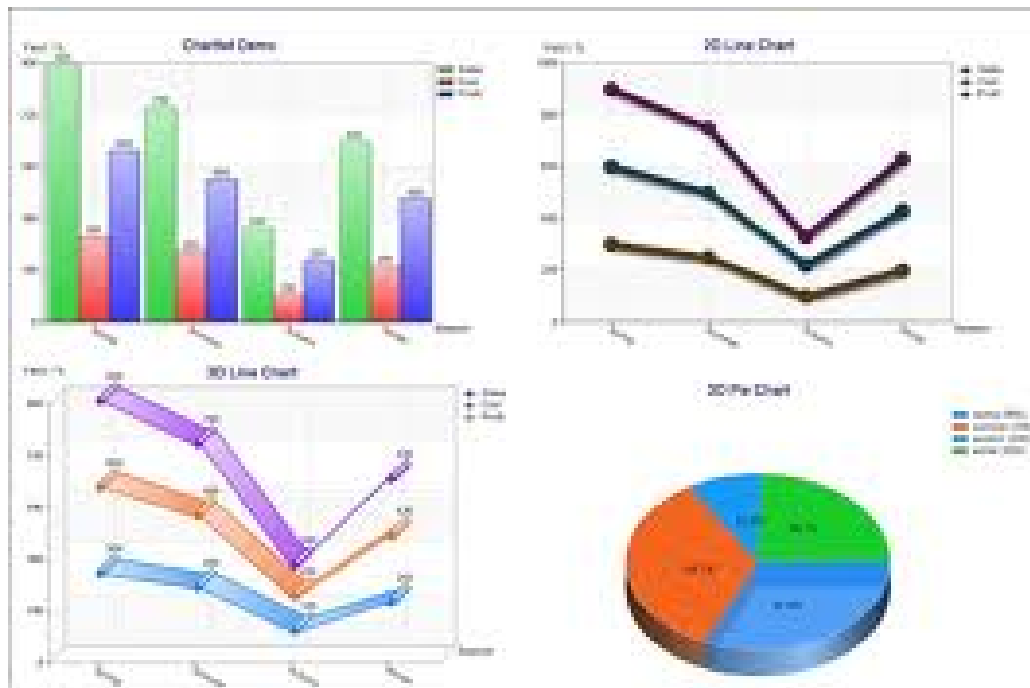
B. 75

C. 65

D. 55

Data Analysis

- Data is often interpreted visually through graphs
- Examine the entire graph – notice labels and headings
- Look for changes – high points, low points, trends
- Some common graphs: line, bar, pie graph, pictograph, stem-and-leaf plot, box-and-whisker plot, scatter plot



Data Analysis

- Example of box-and-whisker plot

A basketball team played 11 games The number of points earned in each game :

23, 25, 27, 29, 30, 34, 38, 40, 41, 42, 45

Median = 34

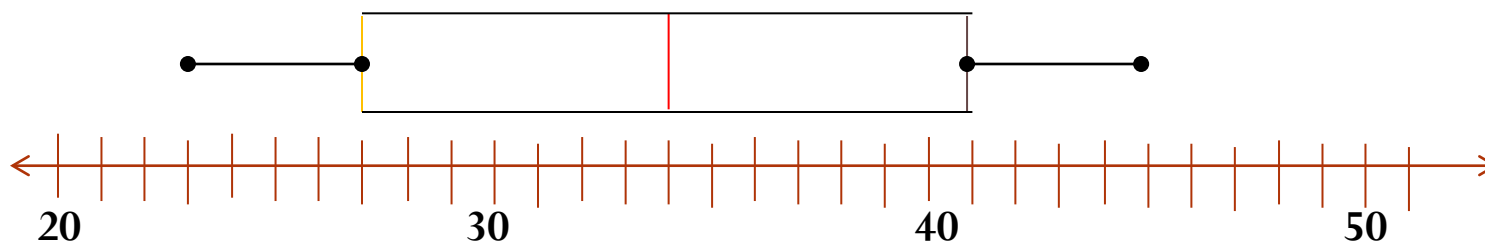
Lower quartile is the median of the lower half of a data set = 27

Upper quartile is the median of the upper half of a data set = 41

Interquartile range = upper quartile – lower quartile = $41 - 27 = 15$

Minimum = 23

Maximum = 45



Probability

- How likely an event may happen

$$\text{Probability of an event happening} = \frac{\text{Number of possible ways it can happen}}{\text{Total of possible ways}}$$

Probability is always between 0 and 1



Probability

- Tossing a Coin

Possible outcomes is Heads or Tails

Can be only one or other at a given time = 1

Total number of possible outcomes is 2

Probability of coin landing on Heads = $\frac{1}{2}$

Probability of coin landing on Tails = $\frac{1}{2}$



- Throwing Dice

Possible outcomes is 1, 2, 3, 4, 5, or 6

Can be only one of six at a given time = 1

Total number of possible outcomes is 6

Probability of dice landing on a 1 = $\frac{1}{6}$



Probability

- Spinner

Possible outcomes is 1, 2, 3, 4, 5, 6, 7, or 8

Total number of outcomes is 8

Can be only one or other at a given time = 1

Example

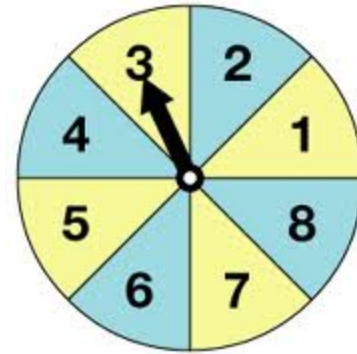
Using the spinner what is probability of spinning either a 5 or a 6

Possible outcome/total outcome

Spinning a 5 = $1/8$

Spinning a 6 = $1/8$

Spinning a 5 or a 6 = $1/8 + 1/8 = 1/4$



*When two events are independent of each other, you **ADD** to find the possible outcome of **either** event occurring and you **MULTIPLY** to find the possible outcome of **both** events occurring.*

Probability

Example

There are 20 children in the cast of a class play, and 8 of the children are boys. Of the boys, 4 have a speaking part in the play, and of the girls, 8 do not have a speaking part in the play. If a child from the cast of the play is chosen at random, what is the probability that the child has a speaking part?

Children	Speaking	Non-speaking	Totals
Boys	4	$8 - 4 = 4$	8
Girls	$12 - 8 = 4$	8	$20 - 8 = 12$
Totals	$4 + 4 = 8$	$4 + 8 = 12$	20

create a table with the separate categories

Fill in the known information. Calculate unknown

$$\begin{aligned}\text{Probability choosing a child with speaking part} &= \frac{\text{number of children with speaking parts}}{\text{total number of children in the play}} \\ &= 8/20 \\ &= 2/5\end{aligned}$$

Sources

- *Math Review for Standardized Tests*
- *www.mathisfun.com*
- *www.shmoop.com*
- *www.testpreview.com*
- *The College Board*
- *www.physicstutors.com*