1. Solve the equation $10z^2 = z + 3$ by factoring.

(a) $z = \frac{2}{5}$ or $\frac{4}{3}$ (b) $z = \frac{3}{5}$ or $-\frac{1}{5}$ (c) $z = \frac{3}{5}$ or $-\frac{1}{2}$ (d) $z = -\frac{3}{5}$ or $\frac{2}{5}$ (e) $z = \frac{1}{3}$ or $-\frac{2}{3}$

Answer: (c)

$10z^2 = z + 3 \Leftrightarrow 10z^2 - z - 3 = 0 \Leftrightarrow (5z-3)(2z+1) = 0$. So $z = \frac{3}{5}$ or $z = -\frac{1}{2}$.

2. Solve the quadratic equation $2x^2 + 12x + 1 = 0$ by completing the square.

(a) $x = -3 \pm \sqrt{\frac{17}{2}}$ (b) $x = -2 \pm \sqrt{\frac{13}{2}}$ (c) $x = 3 \pm \sqrt{\frac{2}{3}}$ (d) $x = -4 \pm \sqrt{\frac{11}{2}}$ (e) $x = -3 \pm \sqrt{\frac{14}{3}}$

Answer: (a)

$2x^2 + 12x + 1 = 0 \Leftrightarrow x^2 + 6x + 3^2 = -\frac{1}{2} + 3^2 \Leftrightarrow (x+3)^2 = \frac{17}{2} \Leftrightarrow x + 3 = \pm\sqrt{\frac{17}{2}}$. So $x = -3 \pm \sqrt{\frac{17}{2}}$.

3. Find all real solutions of the equation $w(w-2) = 3$.

(a) $w = 2$ or 4 (b) $w = -2$ or -1 (c) $w = -1$ or 3 (d) $w = 0$ or 2 (e) $w = \pm 2$

Answer: (c)

$w(w-2) = 3 \Leftrightarrow w^2 - 2w - 3 = 0 \Leftrightarrow (w-3)(w+1) = 0 \Leftrightarrow w = 3$ or $w = -1$

4. Find all real solutions of the equation $\frac{2x}{2x-5} - \frac{x-4}{x+7} = 1$.

(a) $x = \frac{3 \pm \sqrt{117}}{4}$ (b) $x = \frac{5 \pm \sqrt{241}}{16}$ (c) $x = \frac{15 \pm \sqrt{221}}{16}$ (d) $x = \frac{1 \pm \sqrt{10}}{4}$ (e) $x = \frac{9 \pm \sqrt{111}}{2}$

Answer: (e)

$\frac{2x}{2x-5} - \frac{x-4}{x+7} = 1 \Leftrightarrow 2x(x+7) - (x-4)(2x-5) \Leftrightarrow 2x^2 + 14x - 2x^2 + 13x - 20 = 2x^2 + 9x - 35 \Leftrightarrow$

$2x^2 - 18x - 15 = 0 \Leftrightarrow x = \frac{18 \pm \sqrt{324 + 120}}{4}$. So $x = \frac{9 \pm \sqrt{111}}{2}$.

5. For the quadratic equation $2x^2 + kx - 4 = 0$ find the value(s) of k that will ensure that $x = -\frac{2}{3}$ and 3 are the solutions of the quadratic equation.

Answer:

$2x^2 + kx - 4 = 0$. For $x = -\frac{2}{3}$: $2\left(-\frac{2}{3}\right)^2 - \frac{2}{3}k - 4 = 0 \Rightarrow k = -\frac{28}{9}\left(\frac{3}{2}\right) \Rightarrow k = -\frac{14}{3}$. For $x = 3$: $2 \cdot 3^2 + 3k - 4 = 0$ $\Rightarrow$ $k = -\frac{14}{3}$. Thus the value of k is $-\frac{14}{3}$.

6. Using the discriminate, determine how many real solutions the equation $3x^2 = 5 - 6x$ will have, without solving the equations.

Answer:
$3x^2 = 5 - 6x \Leftrightarrow 3x^2 + 6x - 5 = 0, \Delta = b^2 - 4ac = 36 + 60 > 0 \quad \Rightarrow \quad$ 2 real solutions.

7. Find all values for k that ensure that the equation $2kx^2 + 18x + k = 0$ has exactly one solution.

Answer:
$2kx^2 + 18x + k = 0, \mathrm{D} = b^2 - 4ac = 18^2 - 8k^2 = 0.$ Thus there is exactly one solution when

$18^2 - 8k^2 = 0 \Leftrightarrow k = \pm\frac{18}{\sqrt{8}} \Leftrightarrow k = \pm\frac{9\sqrt{2}}{2}.$

8. Evaluate the expression $(7-3i)+(4+9i)$ and write the results in the form $a + bi$.

(a) $3 + 14i$ (b) $6 - 2i$ (c) $12 - 8i$ (d) $14 + 16i$ (e) $11 + 6i$

Answer: (e)
$(7-3i)+(4+9i) = (7+4)+(-3i+9i) = 11 + 6i$

9. Evaluate the expression $(5-i)-(6+3i)$ and write the results in the form $a + bi$.

(a) $-2+4i$ (b) $1-6i$ (c) $-1-4i$ (d) $12-4i$ (e) $3+4i$

Answer: (c)
$(5-i)-(6+3i) = (5-6)+(-i-3i) = -1-4i$

10. Evaluate the expression $(4 - 7i)(1 + 3i)$ and write the results in the form $a + bi$.

(a) $5 + 13i$ (b) $3 + 2i$ (c) $25 + 5i$ (d) $6 - 2i$ (e) $13 - 5i$

Answer: (c)
$(4-7i)(1+3i) = 4 + 12i - 7i - 21i^2 = 4 + 5i + 21 = 25 + 5i$

11. Evaluate the expression $\frac{1}{2+i}$ and write the results in the form $a + bi$.

(a) $\frac{1}{3}+\frac{2}{3}i$ (b) $\frac{1}{5}+\frac{2}{5}i$ (c) $\frac{4}{3}-\frac{2}{3}i$ (d) $\frac{1}{3}-\frac{2}{3}i$ (e) $\frac{2}{5}-\frac{1}{5}i$

Answer: (e)
$\frac{1}{2+i} = \frac{1}{2+i}\cdot\frac{2-i}{2-i} = \frac{2-i}{4-i^2} = \frac{2-i}{5} = \frac{2}{5}-\frac{1}{5}i$

12. Evaluate the expression $\frac{5}{5-2i}$ and write the result in the form *a* + *bi*.

(a) $\frac{3}{23}+\frac{11}{17}i$ (b) $\frac{25}{29}+\frac{10}{29}i$ (c) $\frac{6}{13}+\frac{11}{43}i$ (d) $\frac{17}{19}+\frac{67}{75}i$ (e) $\frac{12}{13}+\frac{2}{27}i$

Answer: (b)

$$\frac{5}{5-2i}=\frac{5(5+2i)}{(5-2i)(5+2i)}=\frac{25+10i}{25-4i^2}=\frac{25+10i}{29}=\frac{25}{29}+\frac{10}{29}i$$

13. Evaluate the expression i^{1828} and write the results in the form *a* + *bi*.

(a) $-i$ (b) i (c) -1 (d) 1 (e) 956

Answer: (d)

$$i^{1828}=\left(i^2\right)^{914}=(-1)^{914}=1$$

14. Evaluate the expression $\frac{\sqrt{-7}\sqrt{-49}}{\sqrt{28}}$ and write the results in the form *a* + *bi*.

Answer:

$$\frac{\sqrt{-7}\sqrt{-49}}{\sqrt{28}}=\frac{i\sqrt{7}\cdot 7i}{2\sqrt{7}}=\frac{7i^2}{2}=-\frac{7}{2}$$

15. Find all solutions of the equation $2x^2 + 3 = 2x$ and express them in the standard form *a* + *bi*.

Answer:

$$2x^2+3=2x \Leftrightarrow 2x^2-2x+3=0 \Leftrightarrow x=\frac{2\pm\sqrt{(-2)^2-4(2)(3)}}{2(2)}=\frac{2\pm\sqrt{-20}}{4}=\frac{1}{2}\pm\frac{\sqrt{5}}{2}i$$

16. Find all real solutions of the equation $2x^3 + x^2 - 4x - 2 = 0$.

(a) $x=-2$ or $1 \pm\sqrt{5}$ (b) $x=-\frac{1}{3},\frac{1}{2}$, or $\sqrt{3}$ (c) $x=-\frac{1}{2}$ or $\pm\sqrt{2}$ (d) $x=\frac{2}{3},\frac{4}{3}$ or 2 (e) $x=\pm 2$ or $\frac{2}{3}$

Answer: (c)

$$2x^3+x^2-4x-2=0 \Leftrightarrow x^2(2x+1)-2(2x+1)=0 \Leftrightarrow (2x+1)\left(x^2-2\right)=0 \Leftrightarrow x=-\frac{1}{2},\pm\sqrt{2}$$

17. Find all real solutions of the equation $\sqrt{2x+1}+\sqrt{x+1}=2$.

(a) $x=-1$ or 2 (b) $x=0$ or $\sqrt{3}$ (c) -1 or 0 (d) $\sqrt{3}$ (e) 0

Answer: (e)

$$\sqrt{2x+1}+\sqrt{x+1}=2 \Leftrightarrow \sqrt{2x+1}=2-\sqrt{x+1} \Leftrightarrow 2x+1=4-4\sqrt{x+1}+x+1 \Leftrightarrow 4\sqrt{x+1}=4-x \Leftrightarrow$$

$16(x+1)=16-8x+x^2 \Leftrightarrow x^2-24x=0 \Leftrightarrow x(x-24)=0$. Now $x=0$ is a solution but $x=24$ is not.

18. Find all real solutions of the equation $x^4 - 5x^2 + 4 = 0$.

(a) $x = \pm 1$ or ± 2 (b) $x = -1$ or 2 (c) $x = -2$ or 1 (d) $x = \pm 1$ or 2 (e) $x = \pm 2$ or 1

Answer: (a)
$x^4 - 5x^2 + 4 = 0 \Leftrightarrow (x^2 - 4)(x^2 - 1) = 0$. So $x = \pm 1$ or ± 2.

19. Find all real solutions of the equation $x - 3\sqrt{x} + 2 = 0$.

(a) $x = 1$ or 2 (b) $x = 1$ or 4 (c) $x = 2$ or 3 (d) $x = \pm 1$ (e) $x = \pm 2$

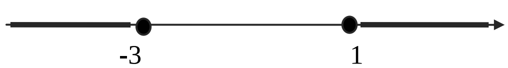
Answer: (b)
$x - 3\sqrt{x} + 2 = 0 \Leftrightarrow (\sqrt{x} - 1)(\sqrt{x} - 2) = 0 \Leftrightarrow \sqrt{x} = 1$ or $2 \Leftrightarrow x = 1$ or 4.

20. Solve the inequality $x^2 < 2x + 8$ in terms of intervals and illustrate the solution set on the real number line.

Answer:
$x^2 < 2x + 8 \Leftrightarrow x^2 - 2x - 8 < 0 \Leftrightarrow (x - 4)(x + 2) < 0$.
Case (i): $x - 4 < 0 \Leftrightarrow x < 4$, and $x+2 > 0 \Leftrightarrow x > -2$; so $-2 < x < 4$.
Case (ii): $x - 4 > 0 \Leftrightarrow x > 4$, and $x+2 < 0 \Leftrightarrow x < -2$, which is impossible. So, the solution is $-2 < x < 4 \Leftrightarrow x \in (-2, 4)$

21. Solve the inequality $5x^2 + 2x \geq 4x^2 + 3$. Express your solution in the form of intervals and illustrate the solution set on the real number line.

Answer:
$5x^2 + 2x \geq 4x^2 + 3 \Leftrightarrow x^2 + 2x - 3 \geq 0 \Leftrightarrow (x - 1)(x + 3) \geq 0$.
Case (i): $x - 1 \geq 0 \Leftrightarrow x > 1$, and $x + 3 \geq 0 \Leftrightarrow x \geq -3$; so $x \geq 1$.
Case (ii): $x - 1 \leq 0 \Leftrightarrow x \leq 1$, and $x + 3 \leq 0 \Leftrightarrow x \leq -3$; so $x \leq -3$
Therefore $x \leq -3$ or $x \geq 1 \Leftrightarrow x \in (-\infty, -3] \cup [1, \infty)$



22. Solve the inequality $\frac{2 + x}{3 - x} \leq 1$ in terms of intervals and illustrate the solution set on the real number line.

Answer:
$\frac{2 + x}{3 - x} \leq 1$.

Case (i): if $3 - x > 0$ (that is, $x < 3$) then $2 + x \leq 3 - x \Leftrightarrow 2x \leq 1 \Leftrightarrow x \leq \frac{1}{2}$; so $x \leq \frac{1}{2}$.

Case (ii): if $3 - x < 0$ (that is, $x > 3$) then $2 + x \geq 3 - x \Leftrightarrow 2x \geq 1 \Leftrightarrow x \leq \frac{1}{2}$; so $x > 3$.

So the solution is $x \leq \frac{1}{2}$ or $x > 3 \Leftrightarrow x \in \left(-\infty, \frac{1}{2}\right] \cup (3, \infty)$

23. Solve the equation $|3x+5|=1$.

(a) $x = -\frac{4}{3}$ or -2 (b) $x = -\frac{5}{3}$ or 1 (c) $x = -\frac{5}{3}$ or -1 (d) $x = \frac{4}{3}$ or 1 (e) $x = \pm\frac{4}{3}$

Answer: (a)

$|3x+5|=1$. So $3x+5=1 \Leftrightarrow 3x=-4 \Leftrightarrow x=-\frac{4}{3}$, or $3x+5=-1 \Leftrightarrow 3x=-6 \Leftrightarrow x=-2$. Thus $x=-\frac{4}{3}$ or $x=-2$.

24. Solve the equation $|x-6|=-1$.

(a) $x = \pm 6$ (b) $x = -5$ (c) $x = 5$ (d) $x = -6$ (e) No solution

Answer: (e)

$|x-6|=-1$. This has no solution for x since $|x-6| \geq 0$ for all x.

25. Solve the equation $4 + 2|x+5| = 5$

(a) $x = -\frac{13}{2}$ or 11 (b) $x = -7$ or -4 (c) $x = -\frac{2}{3}$ or $-\frac{7}{3}$ (d) $x = -\frac{11}{2}$ or $-\frac{9}{2}$ (e) x = ±5

Answer: (d)

$4+2|x+5|=5 \Leftrightarrow |x+5|=\frac{1}{2} \Leftrightarrow x=-\frac{11}{2}$ or $-\frac{9}{2}$.

26. Solve the equation $|x+2|=|3x+1|$.

(a) $x = -\frac{2}{3}$ or $\frac{1}{3}$ (b) $x = -\frac{1}{2}$ or $\frac{5}{2}$ (c) $x = -\frac{3}{4}$ or $\frac{1}{2}$ (d) $x = -\frac{1}{3}$ or $\frac{2}{3}$ (e) $x = -\frac{1}{4}$ or $\frac{3}{4}$

Answer: (c)

$|x+2|=|3x+1|$. So $x + 2 = 3x + 1 \Leftrightarrow x=\frac{1}{2}$, or $x+2=-(3x+1) \Leftrightarrow x+2=-3x-1 \Leftrightarrow x=-\frac{3}{4}$. Therefore the solutions are $x = -\frac{3}{4}$ or $\frac{1}{2}$.

27. Solve the inequality $|x+1| \geq 2$.

Answer:

$|x+1| \geq 2$. So $x+1 \geq 2 \Leftrightarrow x \geq 1$, or $x+1 \leq -2 \Leftrightarrow x \leq -3$. So SS = $\{x | x \leq -3 \text{ or } x \geq 1\}$

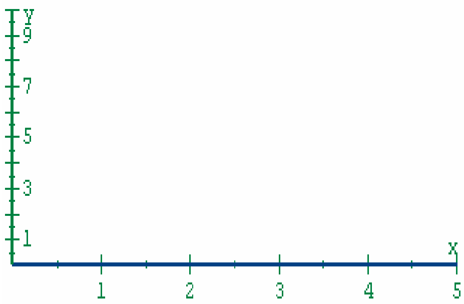
28. Solve the inequality $|3x-4| < 5$.

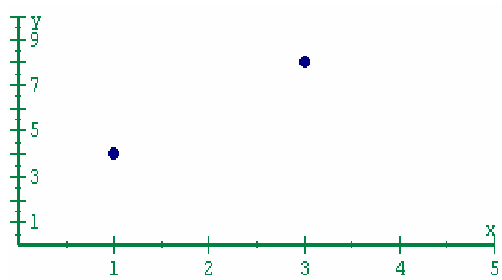
Answer:

$|3x-4|<5 \Leftrightarrow -5<3x-4<5 \Leftrightarrow -1<3x<9 \Leftrightarrow -\frac{1}{3}<x<3$. So SS = $\{x | -\frac{1}{3} < x < 3\}$

29. For the points (1, 4) and (3, 8):
   (a) Plot the points on a coordinate plane.
   (b) Find the distance between them.
   (c) Find the midpoint of the line segment that joins them.

Answer:





(b) $P_1 = (1,4)$ and $P_2 = (3,8)$ so

$$|P_1P_2| = \sqrt{(1-3)^2 + (4-8)^2} = \sqrt{20} = 2\sqrt{5}$$

(c) The midpoint is $\left(\frac{1+3}{2}, \frac{4+8}{2}\right) = (2,6)$.

30. Determine which of the points (1, 2), (3,6) and (4.9) are on the graph of the equation $y = \frac{1}{2}x^2 + \frac{3}{2}$.

(a) (1, 2) (b) (1, 2), (3,6) (c) (3, 6) (d) (3, 6), (4, 9) (e) (1, 2), (3, 6), (4, 9)

Answer: (b)

Substitute to find that $2 = \frac{1}{2}\left(1^2\right) + \frac{3}{2}$ and $6 = \frac{1}{2}\left(3^2\right) + \frac{3}{2}$, but $9 \neq \frac{1}{2}\left(4^2\right) + \frac{3}{2}$.

31. Find the *x*- and *y*-intercepts of the graph of $y = x^2 - 16$.

(a) *x*-intercepts ± 3, *y*-intercept 16
(b) *x*-intercept 4, *y*-intercepts ± 16
(c) *x*-intercept 4, *y*-intercepts ± 16
(d) *x*-intercepts ± 4, *y*-intercept –16
(e) *x*-intercept – 4, *y*-intercept –16

Answer: (d)

$y = 0 \Leftrightarrow x^2 = 16 \Leftrightarrow x = \pm 4; x = 0 \Leftrightarrow y = -16$

32. Find the x-intercept and y-intercepts of the graph of $(x + 2y)(x - 2y) = 1$.

(a) *x*-intercept 1, *y*-intercept $1/\sqrt{2}$
(b) *x*-intercepts ± 1, *y*-intercepts $\pm 1\sqrt{2}$
(c) *x*-intercept -1, no *y*-intercept
(d) *x*-intercept 1, no *y*-intercept
(e) *x*-intercepts ± 1, no *y*-intercept

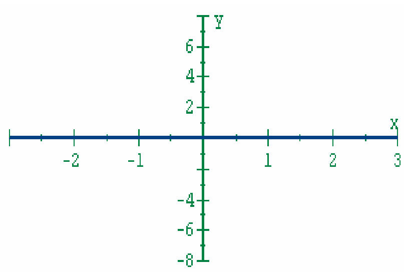
Answer: (e)

$y = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1; x = 0 \Leftrightarrow -4y^2 = 1$, which has no real solution.

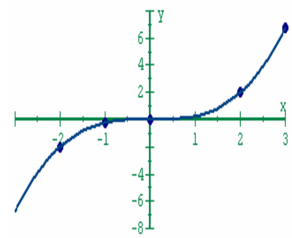
33. Make a table of values and sketch the graph of the equation $4y = x^3$. Find *x*- and *y*-intercepts and test for symmetry.

Answer:

$$4y = x^3 \Leftrightarrow y = x^3/4$$



x	$y = x^3/4$	(x, y)
-2	-2	$(-2,-2)$
-1	$-\frac{1}{4}$	$\left(-1,-\frac{1}{4}\right)$
0	0	$(0, 0)$
2	2	$(2, 2)$
3	$\frac{27}{4}$	$\left(3,\frac{27}{4}\right)$



x-intercept: set $y = 0 \Rightarrow x^3/4 = 0 \Leftrightarrow x = 0$;

y-intercept: set $x = 0 \Rightarrow y = 0$; symmetry:

wrt *x*-axis $-y = x^3/4 \Leftrightarrow y = -x^3/4$ which is changed,

wrt *y*-axis $y = (-x)^3/4 = -x^3/4$ which is changed,

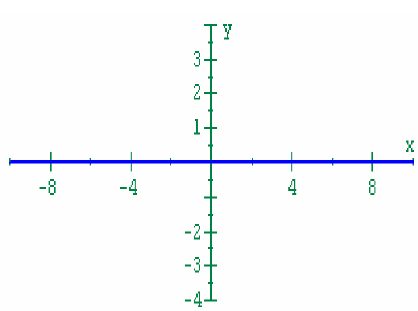
wrt orgin $-y = -x^3/4 \Leftrightarrow y = x^3/4$ which is not changed,

so symmetric wrt orgin.

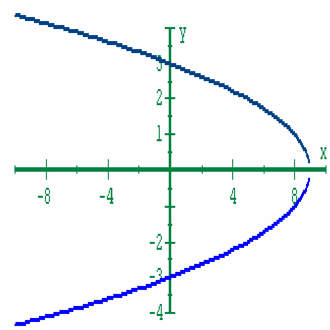
34. Make a table of values and sketch the graph of the equation $x + y^2 = 9$. Find x- and y-intercepts and test for symmetry.

Answer:

$y = \pm\sqrt{9-x}$



x	$y = \pm\sqrt{9-x}$	(x,y)
-7	± 4	$(-7, \pm 4)$
-3	$\pm 2\sqrt{3}$	$(-3,\pm 2\sqrt{3})$
0	± 3	$(0,\pm 3)$
5	± 2	$(5,\pm 2)$
9	0	$(9, 0)$



x – intercept : set $y = 0 \Rightarrow \sqrt{9-x} = 0 \Rightarrow x = 9$;

y – intercept : set $x = 0 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$;

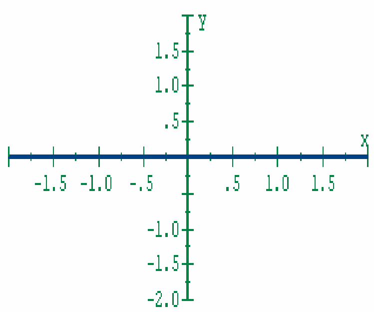
symmetry : wrt x – axis $x + (-y)^2 = 9 \Leftrightarrow x + y^2 = 9$

which is not charged, wrt y – axis $-x + y^2 = 9$

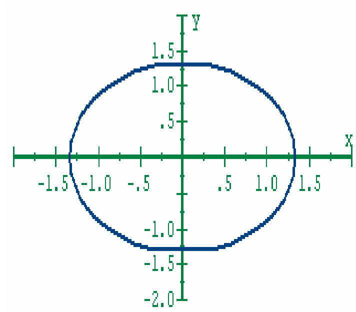
which is changed, wrt orgin $-x + y^2 = 9$ which is changed,

symmetric wrt x – axis.

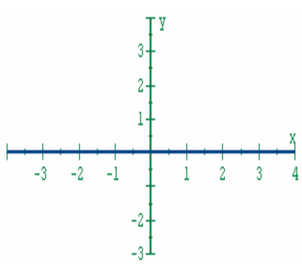
35. Sketch a graph of the equation $9x^2 + 9y^2 = 16$.



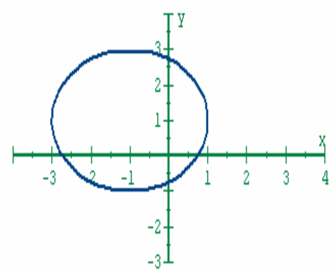
Answer:
$9x^2 + 9y^2 = 16$



36. Sketch a graph of the equation $(x+1)^2 + (y-1)^2 = 4$.



Answer:
$(x+1)^2 + (y-1)^2 = 4$



37. Test the equation $y = x^2 - x^3$ for symmetry.

(a) Symmetric about *x*-axis
(b) Symmetric about *y*-axis
(c) Symmetric about *x*- and *y*-axis
(d) Symmetric about origin
(e) No symmetry

Answer: (e)

38. Test the equation $x^2 y + x^4 y^3 = 1$ for symmetry.

(a) Symmetric about *x*-axis
(b) Symmetric about *y*-axis
(c) Symmetric about *x*- and *y*-axis
(d) Symmetric about origin
(e) No symmetry

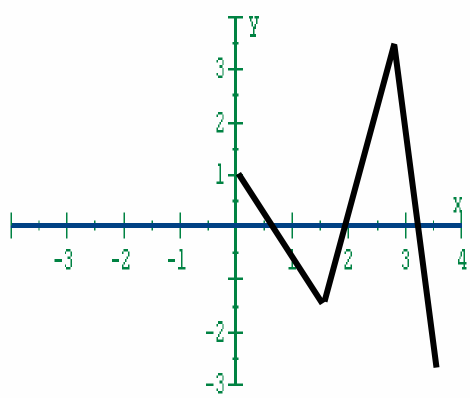
Answer: (b)

39. Test the equation $y = x^5 + x^3$ for symmetry.

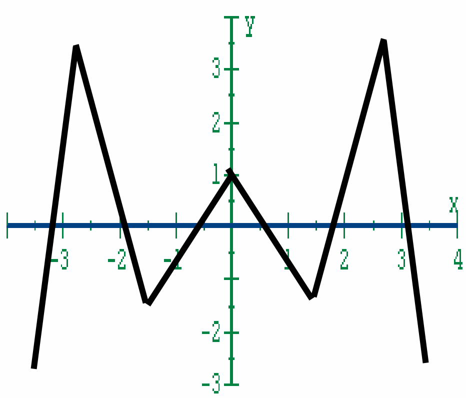
(a) Symmetric about *x*-axis
(b) Symmetric about *y*-axis
(c) Symmetric about *x*- and *y*-axis
(d) Symmetric about origin
(e) No symmetry

Answer: (d)

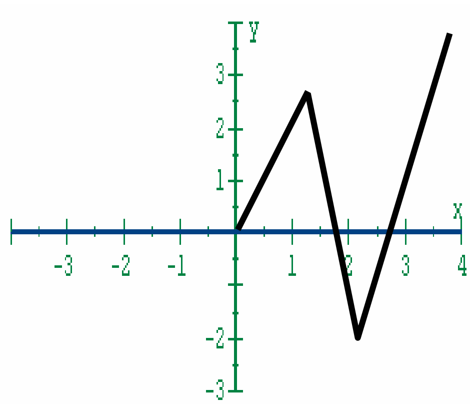
40. Complete the graph, given that it is symmetric about the *y*-axis.



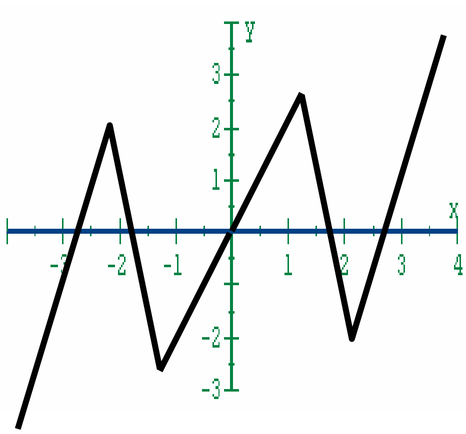
Answer:



41. Complete the graph, given that it is symmetric about the origin.



Answer:



42. Find an equation of the circle with center $(-3,-2)$ and radius 5.

(a) $(x-3)^2+(y+2)^2=25$ (b) $(x+3)^2+(y-2)^2=25$ (c) $(x+3)^2+y^2=25$
(d) $(x+3)^2+(y+2)^2=25$ (e) $x^2+(y-2)^2=25$

Answer: (d)
Using the standard notation, $(h, k) = (-3,-2)$ and $r = 5$. So substituting into $(x-h)^2+(y-k)^2 = r^2$ gives $(x+3)^2+(y+2)^2=25$.

43. Find an equation of the circle that passes through $(-2,-4)$ and has center $(-1, 2)$.

(a) $(x-1)^2+(y-2)^2=\sqrt{2}$ (b) $(x+1)^2+(y+2)^2=36$ (c) $(x+1)^2+y^2=36$
(d) $(x+1)^2+(y-2)^2=37$ (e) $x^2+y^2=37$

Answer: (d)
$C=(-1, 2)$ and $P=(-2,-4)$. Thus $r=|CP|=\sqrt{(-1+2)^2+(2+4)^2}=\sqrt{37}$. Substituting into the standard formula gives $(x+1)^2+(y-2)^2=37$.

44. Find an equation of the circle that satisfies the given conditions: the endpoints of a diameter are $P(-2, 5)$ and $Q(4, 5)$

(a) $(x-2)^2 + (y+1)^2 = 9$ (b) $(x-1)^2 + (y-5)^2 = 9$ (c) $(x+1)^2 + y^2 = 9$

(d) $x^2 + y^2 = 18$ (e) $(x-2)^2 + (y-1)^2 = 18$

Answer: (b)

$P = (-2, 5)$ and $Q = (4, 5)$ are endpoints of a diameter, so the center of the circle is at the midpoint of this diameter: $C = \left(\frac{-2+4}{2}, \frac{5+5}{2}\right) = (1, 5)$. Therfore the radius is $r = |CP| = \sqrt{(1+2)^2 + (5-5)^2} = 3.$

The equation of the circle is $(x-1)^2 + (y-5)^2 = 9.$

45. Show that the equation $16x^2 + 16y^2 + 8x + 24y + 4 = 0$ represents a circle and find the center and radius of the circle.

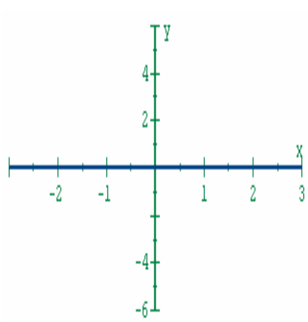
Answer:

$$16x^2 + 16y^2 + 8x + 24y + 4 = 0 \quad \Leftrightarrow \quad \left(x^2 + \frac{1}{2}x\right) + \left(y^2 + \frac{3}{2}y\right) = -\frac{1}{4} \quad \Leftrightarrow$$

$$\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) + \left(y^2 + \frac{3}{2}y + \frac{9}{16}\right) = -\frac{1}{4} + \frac{1}{16} + \frac{9}{16} \quad \Leftrightarrow \quad \left(x + \frac{1}{4}\right)^2 + \left(y + \frac{3}{4}\right)^2 = \frac{3}{8}.$$

This is a circle with center $\left(-\frac{1}{4}, -\frac{3}{4}\right)$ and radius $\frac{\sqrt{6}}{4}$.

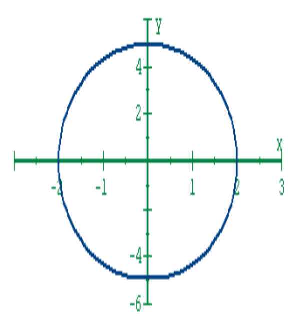
46. $25x^2 + 4y^2 = 100$. Determine the lengths of the major and minor axes, and sketch the graph.



Answer:

$$25x^2 + 4y^2 = 100 \Rightarrow \frac{x^2}{4} + \frac{y^2}{25} = 1 \Rightarrow$$

major axis has length $2a = 10$, minor axis has length $2b = 4$.



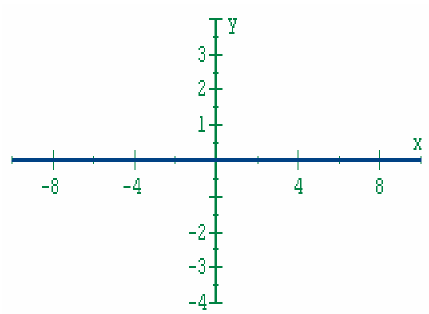
47. Find the vertices and asymptotes of the hyperbola $x^2 - 6y^2 - 18 = 0$, and sketch its graph.

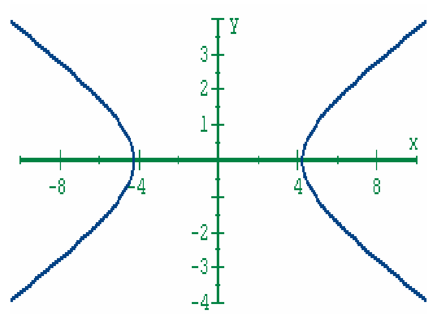
Answer:

$$x^2 - 6y^2 - 18 = 0 \quad \Rightarrow \quad \frac{x^2}{18} - \frac{y^2}{3} = 1 \quad \Rightarrow$$

$a = 3\sqrt{2}$, $b = \sqrt{3}$, so the vertices are $\left(\pm 3\sqrt{2}, 0\right)$

and the asymptotes are $y = \pm \frac{\sqrt{6}}{6} x$.





48. Find the slope of the line through $P(-3, 5)$ and $Q(7, 7)$.

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{11}{2}$

Answer: (d)

$$\text{slope} = \frac{7-5}{7-(-3)} = \frac{1}{5}$$

49. Find an equation of the line with slope $-\frac{2}{3}$ that passes through $(-1, 2)$.

(a) $2x + y - 1 = 0$ (b) $x + 2y - 3 = 0$ (c) $2x + 3y - 4 = 0$
(d) $x + 3y - 2 = 0$ (e) $4x + 2y + 4 = 0$

Answer: (c)

$$m = -\frac{2}{3}, \text{ and } (x_1, y_1) = (-1, 2) \ \Rightarrow \ y - y_1 = m(x - x_1) \ \Rightarrow \ y - 2 = -\frac{2}{3}(x+1) \ \Leftrightarrow \ 2x + 3y - 4 = 0$$

50. Find an equation of the line that passes through $(-3, -1)$ and $(2, 4)$.

(a) $x - 2y + 1 = 0$ (b) $x - y + 2 = 0$ (c) $3x - 2y - 1 = 0$
(d) $3x - 3y + 1 = 0$ (e) $4x + 2y = 0$

Answer: (b)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-1)}{2-(-3)} = 1 \ \Rightarrow \ y - y_1 = m(x - x_1) \ \Rightarrow \ y + 1 = 1(x+3) \ \Leftrightarrow \ x - y + 2 = 0$$

51. Find an equation of the line with slope $\frac{2}{3}$ and y-intercept 6.

(a) $x - y - 12 = 0$ (b) $3x - 2y + 1 = 0$ (c) $2x + 3y - 12 = 0$
(d) $3x + 2y + 4 = 0$ (e) $2x - 3y + 18 = 0$

Answer: (e)

$m = \frac{2}{3}$, $b = 6$. Substituting into $y = mx + b$ gives $y = \frac{2}{3}x + 6 \ \Leftrightarrow \ 2x - 3y + 18 = 0$

52. Find an equation of the line with x-intercept -2 and y-intercept 4.

(a) $3x-2y+1=0$ (b) $2x-y+4=0$ (c) $x+y+2=0$
(d) $2x+2y+3=0$ (e) $x-6y+2=0$

Answer: (b)
Because the x intercept is -2, $(-2, 0)$ is a point on the line, and similarly for the y intercept, $(0, 4)$ is a point on the line. Thus $m=\frac{4-0}{0+2}=2$ and substituting into $y-y_1=m(x-x_1)$ gives $y-0=2(x+2) \Leftrightarrow 2x-y+4=0$.

53. Find an equation of the line parallel to the y axis that passes through $(2, 3)$.

(a) $x=3$ (b) $x=1$ (c) $y=3$ (d) $x=2$ (e) $y=1$

Answer: (d)
The line passes through the point (2, 3) and has an undefined slope (since the line is parallel to the y-axis.) Hence, the abscissa of the line is a constant 2 and the ordinate is arbitrary. Therefore, the equation of the line is $x=2$.

54. Find an equation of the line with y-intercept 3 and that is parallel to the line $x+2y+5=0$.

(a) $x+2y-6=0$ (b) $2x+2y-3=0$ (c) $3x-y-2=0$
(d) $3x+4y-1=0$ (e) $x+y-2=0$

Answer: (a)
$x+2y+5=0 \Leftrightarrow 2y=-(x+5) \Leftrightarrow y=-\frac{1}{2}(x+5)$ which is a line with slope $m=-\frac{1}{2}$. Since the unknown line is parallel to this line, it also has slope $m=-\frac{1}{2}$. Substituting for m and $b=3$ into $y=mx+b$

gives $y=-\frac{1}{2}x+3 \Leftrightarrow x+2y-6=0$

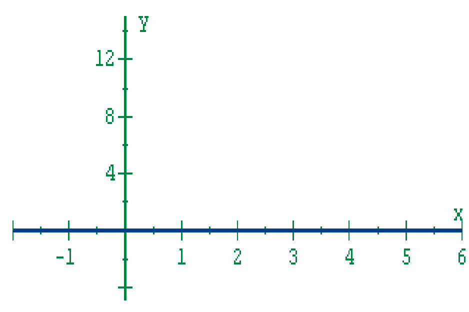
55. Find an equation of the line that is perpendicular to the line $2x-3y=1$ and that passes through $\left(\frac{1}{4}, -\frac{3}{5}\right)$.

(a) $6x+4y-1=0$ (b) $6x-4y-1=0$ (c) $3x+2y-9=0$
(d) $8x+y-3=0$ (e) $60x+40y+9=0$

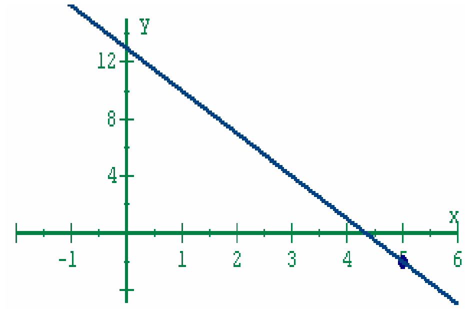
Answer: (e)
$2x-3y=1 \Leftrightarrow 3y=2x-1 \Leftrightarrow y=\frac{2}{3}x-\frac{1}{3}$. Since our unknown line is perpendicular to this line it must have slope $m=-\frac{3}{2}$ and, in addition, it passes through the point $\left(\frac{1}{4},-\frac{3}{5}\right)$. Substituting into $y-y_1=m(x-x_1)$ gives $y+\frac{3}{5}=-\frac{3}{2}\left(x-\frac{1}{4}\right) \Leftrightarrow y+\frac{3}{5}=-\frac{3}{2}x+\frac{3}{8} \Leftrightarrow 60x+40y+9=0$.

56.(a) Sketch the line with slope –3 that passes through the point $(5,-2)$.

(b) Find the equation of this line.



Answer:

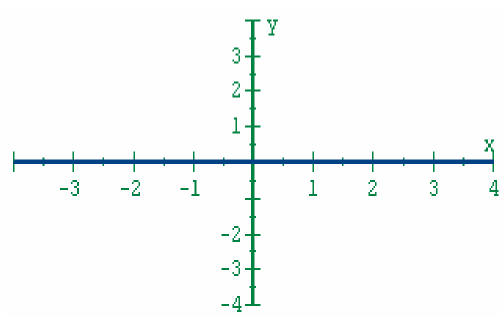


(a)

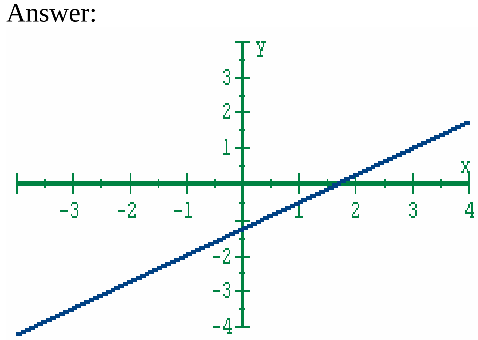
(b) Substituting $(x_1, y_1) = (5,-2)$ and $m = -3$ into $y - y_1 = m(x - x_1)$ gives $y + 2 = -3(x - 5)$

$\Leftrightarrow\ y + 2 = -3x + 15\ \Leftrightarrow\ 3x + y - 13 = 0$

57. Find the slope and *y*-intercept of the line $3x - 4y = 5$ and draw its graph.

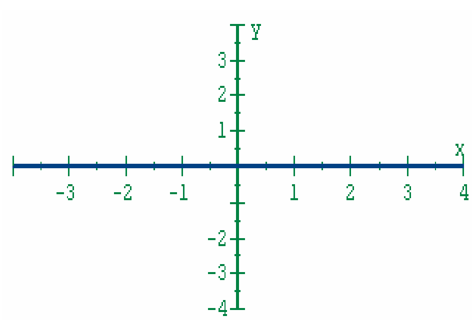


Answer:

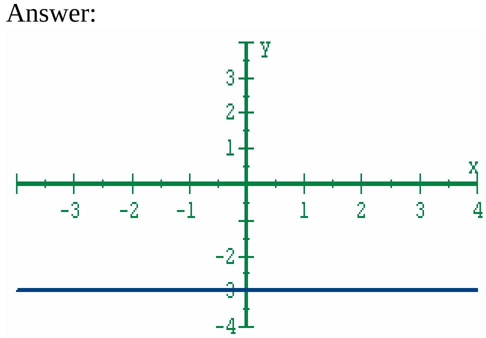


$3x - 4y = 5\ \Leftrightarrow\ y = \frac{3}{4}x - \frac{5}{4}\ \Rightarrow\ m = \frac{3}{4}$ and $b = -\frac{5}{4}$

58. Find the slope and *y* intercept of the line $3y + 9 = 0$ and draw its graph.



Answer:



$3y + 9 = 0\ \Leftrightarrow\ y = -3\ \Rightarrow\ m = 0$ and $b = -3$

59. If $f(x)=x^3+2x-1$, find $f(0)$, $f(3)$, $f(-3)$, $f(-x)$, and $f(1/a)$.

Answer:
$f(x)=x^3+2x-1,\ f(0)=0^3+2\cdot 0-1=-1,\ f(3)=3^3+2\cdot 3-1=27+6-1=32,$
$f(-3)=(-3)^3+2(-3)-1=-27-6-1=-34,\ f(-x)=(-x)^3+2(-x)-1=-x^3-2x-1,$
$f(1/a)=(1/a)^3+2(1/a)-1=1/a^3+2/a-1$

60. Find $f(1),\ f(-1),\ f\left(\frac{1}{3}\right),\ f\left(\frac{1}{3}x\right),\ f(3x),\ f(x^2)$ *and* $[f(x)]^2$ given that $f(x)=4x+1$

Answer:
$f(x)=4x+1,\ \ f(1)=4\cdot 1+1=5,\ f(-1)=4\cdot(-1)+1=-3,\ f\left(\frac{1}{3}\right)=4\cdot\frac{1}{3}+1=\frac{7}{3},\ f\left(\frac{1}{3}x\right)=4\left(\frac{1}{3}x\right)+1=$

$\frac{4}{3}x+1,\ f(3x)=4(3x)+1=12x+1,\ f(x^2)=4x^2+1,\ [f(x)]^2=(4x+1)^2=16x^2+8x+1$

61. For the function $f(x)=2x^2-x+1$ find $f(a)$, $f(a)+f(h)$, $f(a+h)$, and $\frac{f(a+h)-f(a)}{h}$, where a and h are real numbers and $h\neq 0$.

Answer:

$f(x)=2x^2-x+1,\ \ f(a)=2a^2-a+1,\ f(a)+f(h)=2a^2-a+1+2h^2-h+1=2a^2-a+2h^2-h+2,$
$f(a+h)=2(a+h)^2-(a+h)+1=2a^2+4ah+2h^2-a-h+1,$
$$\frac{f(a+h)-f(a)}{h}=\frac{2a^2+4ah+2h^2-a-h+1-\left(2a^2-a+1\right)}{h}=4a+2h-1$$

62. For the function $f(x)=\frac{x+1}{x}$ find $f(2+h)$, $f(x+h)$, and $\frac{f(x+h)-f(x)}{h}$, where $h\neq 0$.

Answer:
$f(x)=\frac{x+1}{x},\ f(2+h)=\frac{2+h+1}{2+h}=\frac{3+h}{2+h},\ f(x+h)=\frac{x+h+1}{x+h},$
$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{x+h+1}{x+h}-\frac{x+1}{x}}{h}=\frac{x(x+h+1)-(x+h)(x+1)}{x(x+h)h}=\frac{x^2+xh+x-x^2-xh-x-h}{x(x+h)h}=-\frac{1}{x(x+h)}$$

63. Find the domain and range of the function $f(x)=2x^2+1, -1\le x\le 2$.

(a) Domain $[-2,2]$, Range $[1,9]$
(b) Domain $[-1,1]$, Range $[2,9]$,
(c) Domain $[-1,2]$, Range $[4,9]$
(d) Domain $[-1,2]$, Range $[2,\infty]$
(e) Domain $[-1,2]$, Range $[1,9]$

Answer: (e)
$f(x)=2x^2+1, -1\le x\le 2 \quad\Leftrightarrow\quad 0\le x^2\le 4 \quad\Leftrightarrow\quad 1\le 2x^2+1\le 9.$ Then the domain is $[-1,2]$ and the range is $[1,9]$.

64. Find the domain and range of the function $g(x)=\sqrt{6-4x}$.

(a) Domain $[-4,4]$, Range $[2,\infty)$ (b) Domain $(-\infty,3]$ Range $[3,\infty)$

(c) Domain $\left(-\infty,\frac{3}{2}\right]$, Range $[0,\infty)$ (d) Domain $(-\infty,3]$ Range $[0,\infty)$

(e) Domain $[4,6]$, Range $[0,\infty)$

Answer: (c)

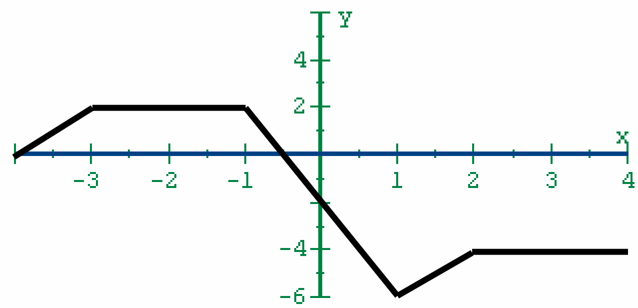
$g(x)=\sqrt{6-4x}$. Since $6\text{-}4x\ge 0 \Leftrightarrow 4x\le 6 \Leftrightarrow x\le\frac{3}{2}$, the domain is $\left(-\infty,\frac{3}{2}\right]$ and the range is $[0,\infty)$.

65. Find the domain of the function $f(x)=\frac{4}{3x-7}$.

Answer:

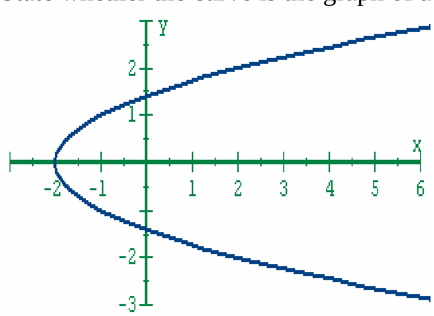
$f(x)=\frac{4}{3x-7}$. Since $3x-7\neq 0 \Leftrightarrow 3x\neq 7 \Leftrightarrow x\neq\frac{7}{3}$, we have $D=\left\{x \mid x\neq\frac{7}{3}\right\}$.

66. The graph of a function *f* is given.
(a) State the values of *f*(−1), *f*(0), *f*(1), and *f*(3).
(b) State the domain and range of *f*.
(c) State the intervals on which *f* is increasing and on which *f* is decreasing.



Answer:
(a) $f(-1) = 2$, $f(0) = -2$, $f(1) = -6$, $f(3) = -4$
(b) Domain = $[-4,4]$, Range = $[-6,2]$
(c) *f* is increasing on $[-4,-3]$ and $[1,2]$ and decreasing on $[-1,1]$.

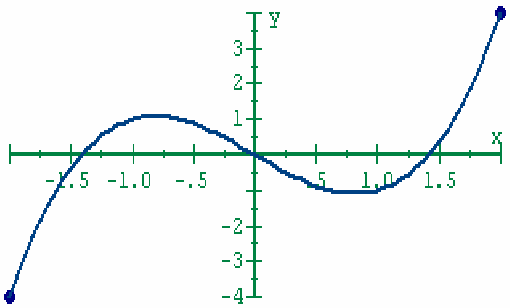
67. State whether the curve is the graph of a function of *x*. If it is, state the domain and range of the function.



(a) Function, domain $[-1,5)$, range $(-2.5,2.5)$
(b) Function, domain $(-2.5,2.5)$, range $[-1,5)$,
(c) Function, domain $(-1,5)$, range $[-2.5,2.5]$
(d) Function, domain $[-2.5,2.5]$ range $[-1,5]$
(e) Not a function

Answer: (e)
The curve is not the graph of a function because it fails the vertical line test.

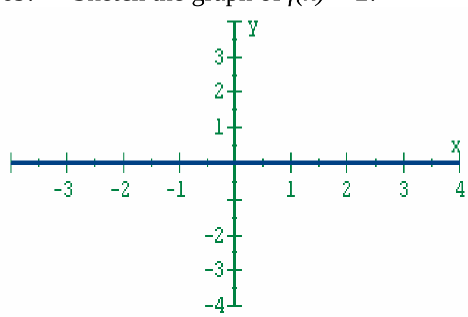
68. State whether the curve is the graph of a function of *x*. If it is, state the domain and range of the function.



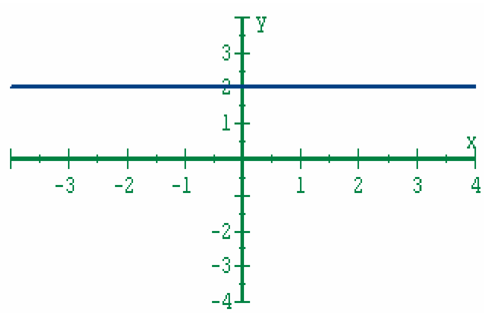
(a) Function, domain $[-2,2)$, range $(-4,4)$
(b) Function, domain $(-2,2)$, range $[-4,4]$
(c) Function, domain $(-2,2)$, range $(-4,4)$
(d) Function, domain $(-4,4)$, range $(-2,2)$
(e) Not a function

Answer: (c)

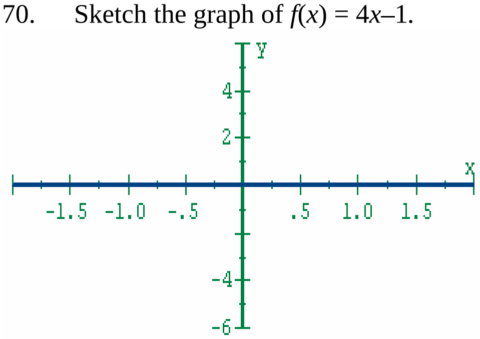
69. Sketch the graph of $f(x) = 2$.



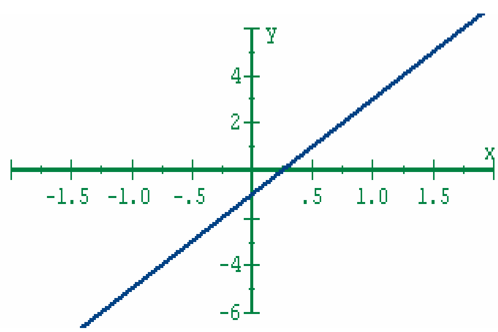
Answer:



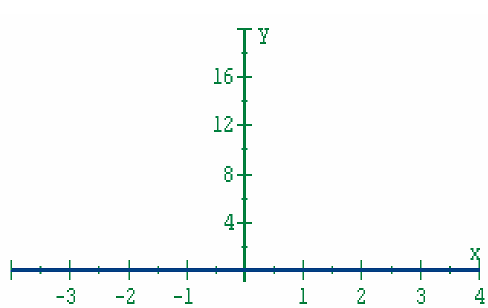
70. Sketch the graph of $f(x) = 4x-1$.



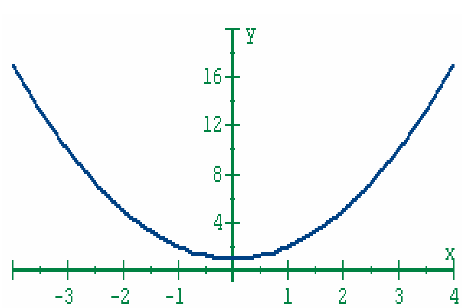
Answer:



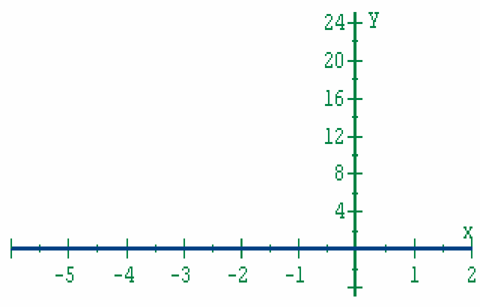
71. Sketch the graph of $f(x) = x^2 + 1$.



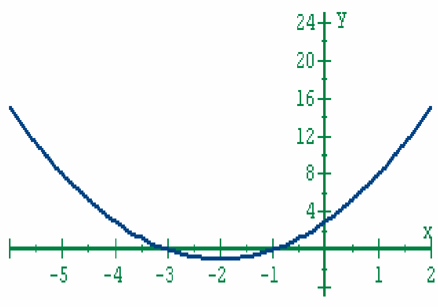
Answer:



72. Sketch the graph of $f(x) = x^2 + 4x + 3$.

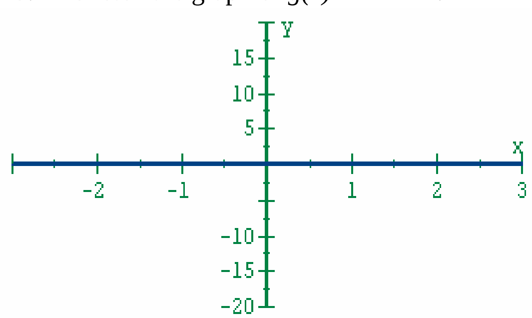


Answer:

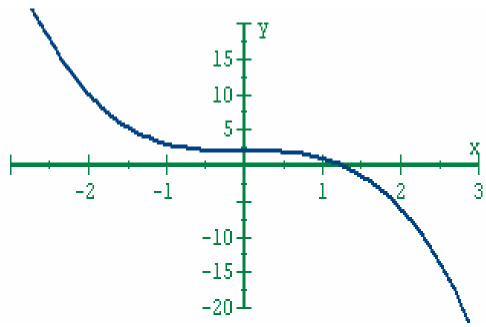


$x^2 + 4x + 3 = (x+2)^2 - 1$

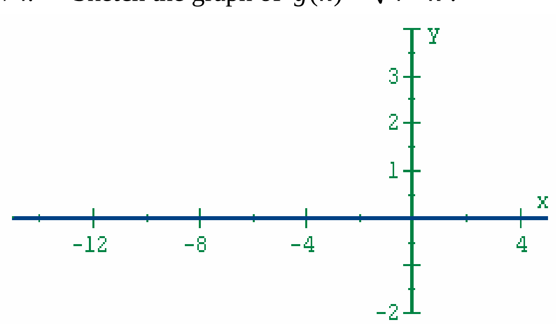
73. Sketch the graph of $g(x) = 2 - x^3$.



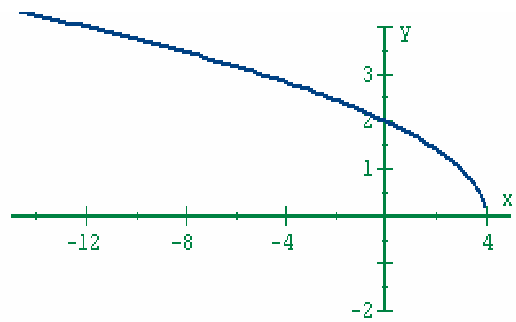
Answer:



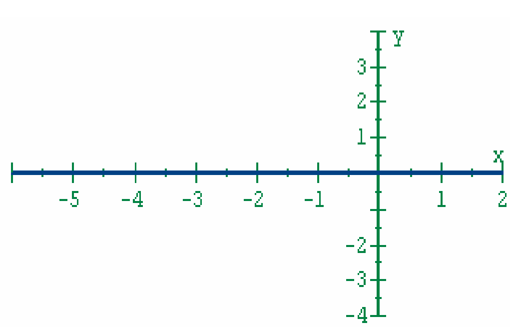
74. Sketch the graph of $g(x) = \sqrt{4 - x}$.



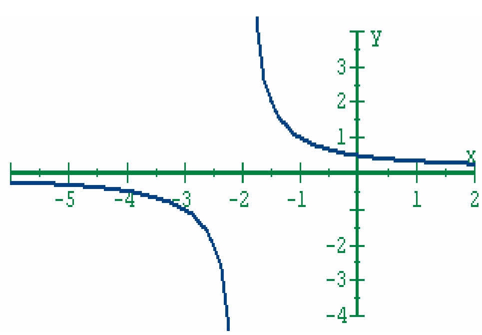
Answer:



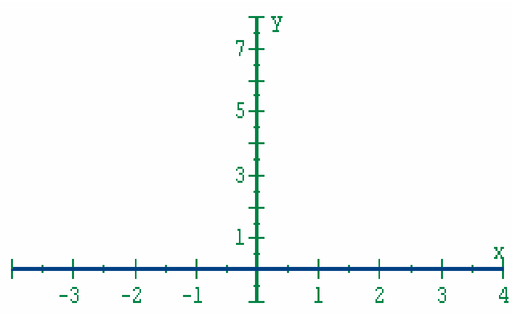
75. Sketch the graph of $F(x) = \dfrac{1}{x+2}$.



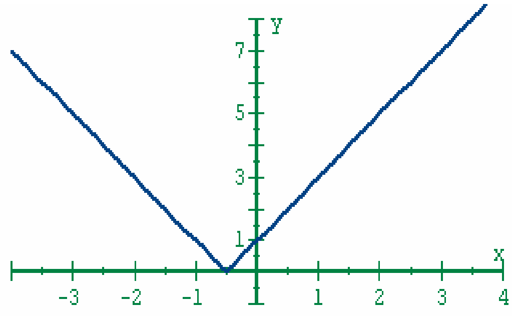
Answer:



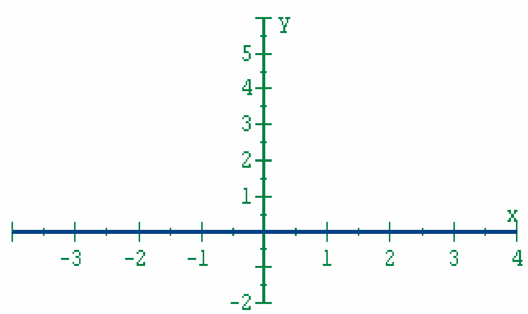
76. Sketch the graph of. $H(x)=|2x+1|$



Answer:



77. Sketch the graph of the piecewise-defined function $f(x)=\begin{cases}-1 \text{ if } x<1 \\ x+1 \text{ if } x\geq 1\end{cases}$



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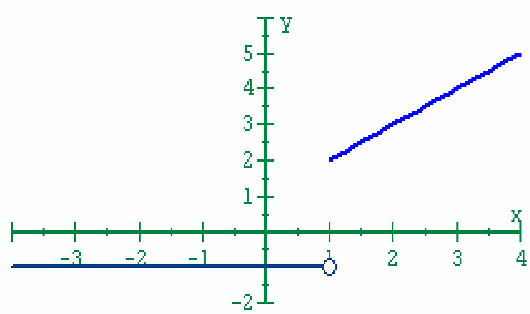


78. Suppose that the graph of *f* is given. Describe how the graph of $y=f(x-4)$ can be obtained from the graph of *f*.

(a) By shifting the graph of *f* 4 units to the left.
(b) By shifting the graph of *f* 4 units down.
(c) By shifting the graph of *f* 4 units up.
(d) By shifting the graph of *f* 4 units to the right.
(e) By reflecting the graph of *f* in the *x*-axis

Answer: (d)

79. Suppose that the graph of *f* is given. Describe how the graph of $y=f(x)+2$ can be obtained from the graph of *f*.

(a) By shifting the graph of *f* 2 units up.
(b) By shifting the graph of *f* 2 units down.
(c) By shifting the graph of *f* 2 units to the left.
(d) By shifting the graph of *f* 2 units to the right.
(e) By reflecting the graph of *f* in the *y*-axis.

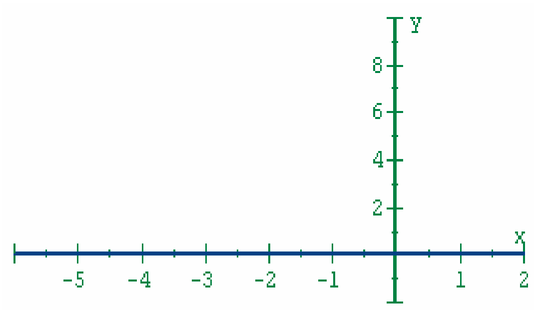
Answer: (a)

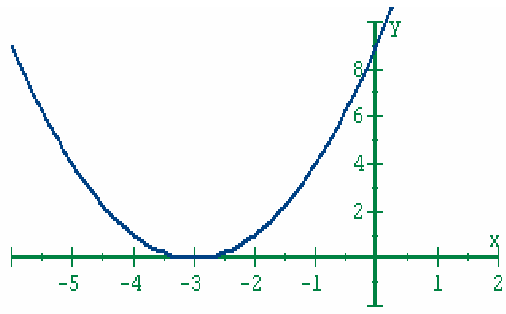
80. Suppose that the graph of *f* is given. Describe how the graph of $y=-f(x)$ can be obtained from the graph of *f*.

(a) By reflecting the graph of *f* in the *x*-axis.
(b) By reflecting the graph of *f* in the *y*-axis.
(c) By reflecting the graph of *f* in the *x* and *y*-axis.
(d) By shifting the graph of *f* 1 unit down.
(e) By shifting the graph of *f* 1 unit to the left.

Answer: (a)

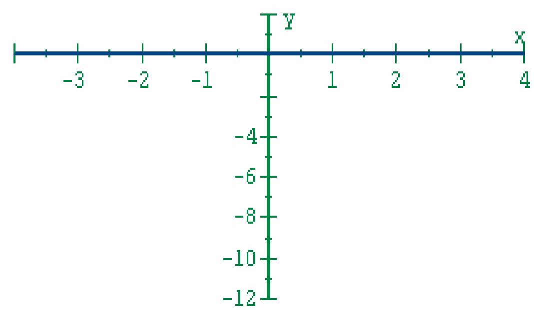
81. Sketch the graph of the function $f(x) = (x+3)^2$ not by plotting points, but by starting with the graphs of standard functions and applying transformations.

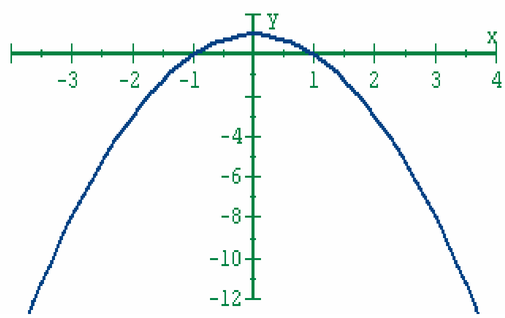




Answer:
$f(x) = (x+3)^2$. Shift $y = x^2$ 3 units to the left.

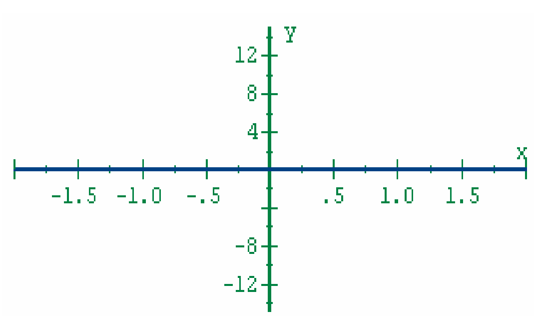
82. Sketch the graph of the function $f(x) = 1 - x^2$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

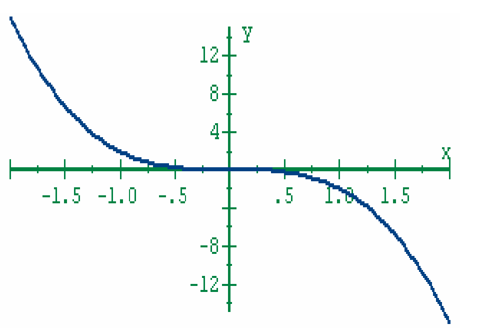




Answer:
$f(x) = 1 - x^2$. Reflect $y = x^2$ in the x-axis and then shift 1 unit upwards.

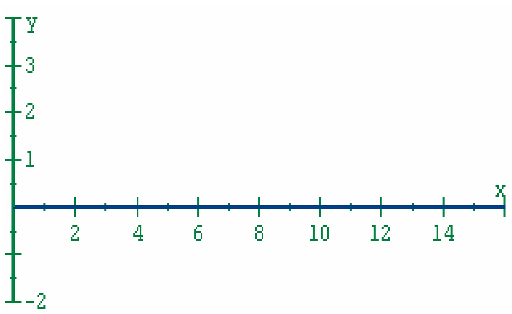
83. Sketch the graph of the function $f(x) = -2x^3$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

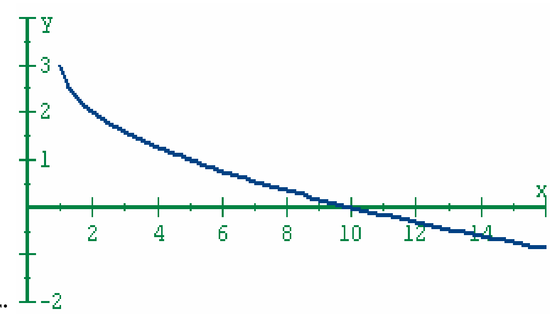




Answer:
$f(x) = -2x^3$. Reflect $y = x^3$ in the x-axis and stretch vertically by a factor of 2.

84. Sketch the graph of the function $y = 3 - \sqrt{x-1}$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

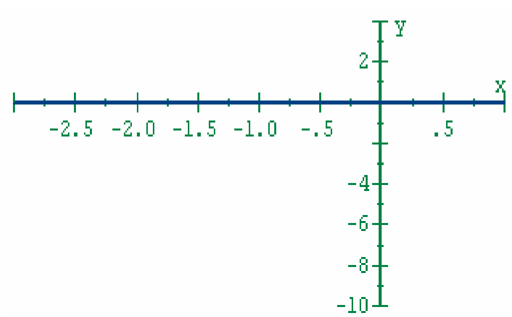


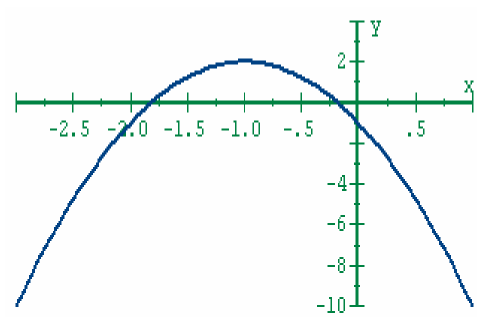


Answer:

$y = 3 - \sqrt{x-1}$. Shift $f(x) = \sqrt{x}$ 1 unit to the right, reflect in the x-axis, and then shift 3 units upwards.

85. Sketch the graph of the function $y = 2 - 3(x+1)^2$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

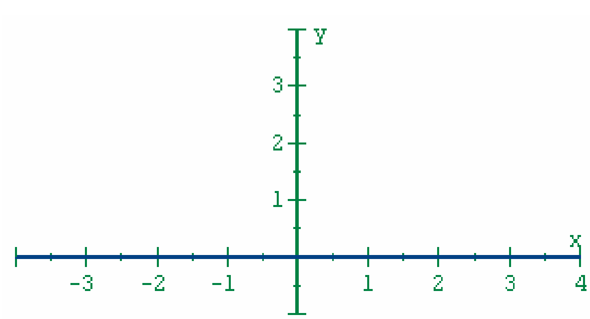


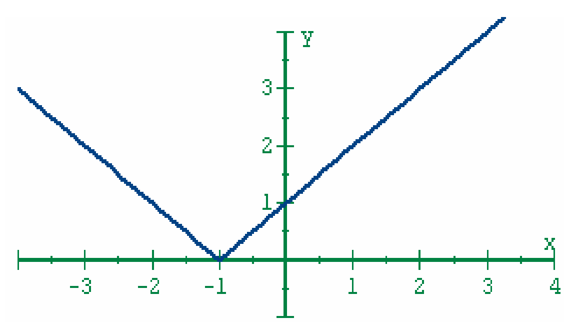


Answer:

$y = 2 - 3(x+1)^2$. Shift $f(x) = x^2$, 1 unit to the left, stretch vertically by a factor of 3, reflect in the x-axis, and then shift 2 units upwards.

86. Sketch the graph of the function $y = |x+1|$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

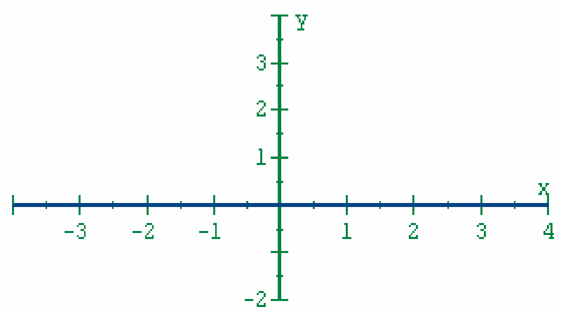


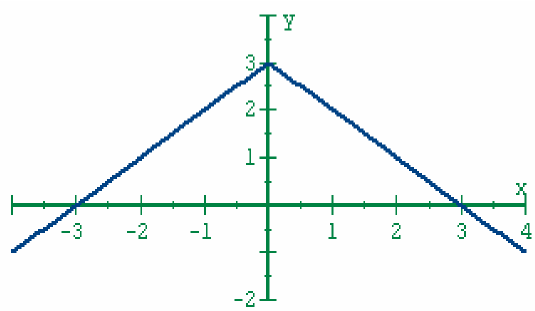


Answer:

$y = |x+1|$. Shift $f(x) = |x|$ 1 unit to the left.

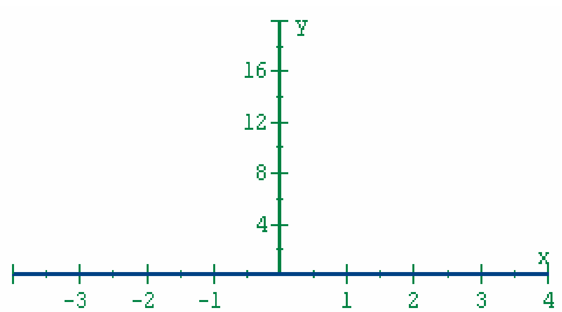
87. Sketch the graph of the function $y = 3 - |x|$, not by plotting points, but by starting with the graphs of standard functions and applying transformations.

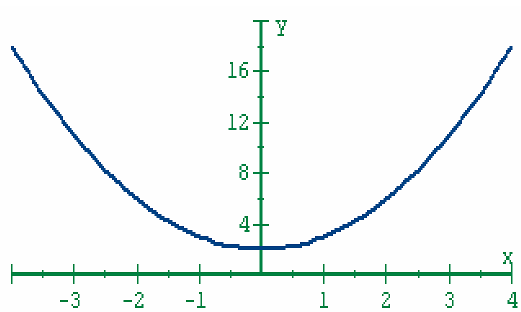




Answer:
$y = 3 - |x|$. Reflect $f(x) = |x|$ in the x-axis and then shift 3 units upwards.

88. Sketch the graph of the parabola $y = x^2 + 2$ and state the coordinates of its vertex and its intercepts.

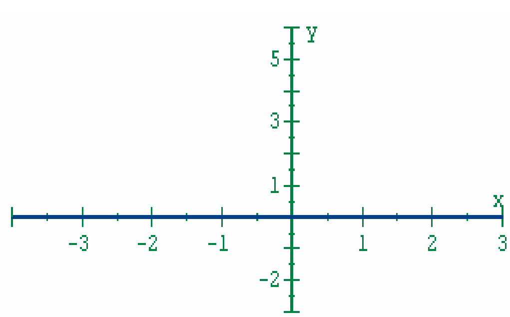


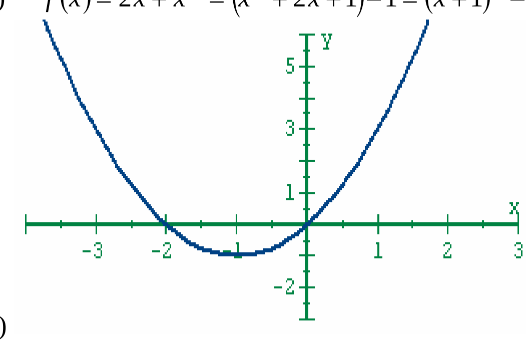


Answer:
The vertex is at $(0,2)$, there is no x-intercept, and the y-intercept is 2.

89. (a) Express the quadratic function $f(x) = 2x + x^2$ in standard form.
(b) Sketch the graph of $f(x)$.
(c) Find the maximum or minimum value of $f(x)$.

Answer:
(a) $f(x) = 2x + x^2 = (x^2 + 2x + 1) - 1 = (x+1)^2 - 1$.





(b)
(c) The minimum value is $f(-1) = -1$.

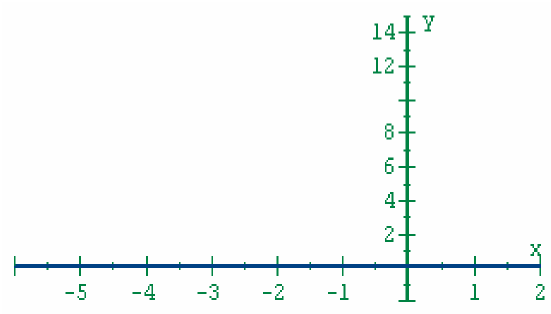
90. (a) Express the quadratic function $f(x) = 5 - 8x - 2x^2$ in standard form.

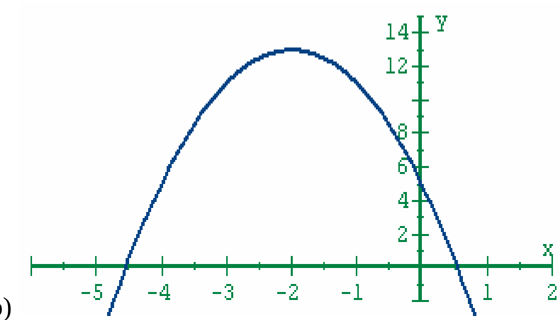
(b) Sketch the graph of $f(x)$.

(c) Find the maximum or minimum value of $f(x)$.

Answer:

(a) $f(x) = 5 - 8x - 2x^2 = -2\left(x^2 + 4x + 4\right) + 13 = -2(x+2)^2 + 13.$





(b)

(c) The maximum value is $f(-2) = 13$.

91. Find the maximum or minimum value of the function $f(x) = 2x^2 + 8x + 11$.

Answer:

$f(x) = 2x^2 + 8x + 11 \Rightarrow a = 2$ and $b = 8,$ so the minimum value is $f\left(-\frac{8}{4}\right) = f(-2) = 3.$

92. Find the maximum or minimum value of the function $f(x) = 3 - 4x - 4x^2$.

Answer:

$f(x) = 3 - 4x - 4x^2 \Rightarrow a = -4$ and $b = -4,$ so the maximum value is $f\left(-\frac{-4}{-8}\right) = f\left(-\frac{1}{2}\right) = 4.$

93. Find the domain and range of the function $f(x) = x^2 - 4x - 2$.

Answer:

$f(x) = x^2 - 4x - 2 = \left(x^2 - 4x + 4\right) - 4 - 2 = (x-2)^2 - 6.$ Then the domain of the function is R and since the minimum value of the function is $f(2) = -6$, the range of the function is the interval $[-6, \infty)$.

94. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$ to evaluate the expression g(f(1)).

(a) −3 (b) −2 (c) −1 (d) 0 (e) 1

Answer: (c)

$f(1) = 2 \cdot 1 - 4 = -2.\ \ g(f(1)) = g(-2) = 3 - (-2)^2 = 3 - 4 = -1$

95. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$ to evaluate the expression g(g(3)).

(a) 11 (b) 3 (c) −12 (d) −33 (e) −40

Answer: (d)

$g(3) = 3 - 3^2 = -6.\ \ g(g(3)) = g(-6) = 3 - (-6)^2 = 3 - 36 = -33$

96. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$ to evaluate the expression $(g \circ f)(-2)$.

(a) -141 (b) -61 (c) 0 (d) 1 (e) 1729

Answer: (b)

$f(-2) = -8.\ (g \circ f)(-2) = g(-8) = 3 - (-8)^2 = -61$

97. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$ to evaluate the expression $(g \circ g)(2)$.

(a) 2 (b) 4 (c) 9 (d) 11(e) 114

Answer: (a)

$g(2) = 3 - 2^2 = -1.\ (g \circ g)(2) = g(-1) = 3 - (-1)^2 = 2$

98. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$ to evaluate the expression $(g \circ f)(x)$.

(a) $-x^2 - x + 13$ (b) $-4x^2 + 16x - 13$ (c) $2x^2 - 3x + 5$ (d) $-5x^2 + 8x - 7$ (e) $3x^2 + 2x + 2$

Answer: (b)

$f(x) = 2x - 4.\ (g \circ f)(x) = g(2x - 4) = 3 - (2x - 4)^2 = 3 - (4x^2 - 16x + 16) = -4x^2 + 16x - 13$

99. Use $f(x) = 2x - 4$ and $g(x) = 3 - x^2$` to evaluate the expression $(g \circ g)(x)$.

(a) $x^4 - 12x^2 - 2$ (b) $x^6 - 4x^4 + 16x^2$ (c) $2x^4 - x^3 + x$ (d) $3x^4 - x^2 + 1$ (e) $-x^4 + 6x^2 - 6$

Answer: (e)

$g(x) = 3 - x^2.\ (g \circ g)(x) = g(3 - x^2) = 3 - (3 - x^2)^2 = 3 - (9 - 6x^2 + x^4) = -x^4 + 6x^2 - 6$

100. Determine whether or not the function $f(x) = x^2 - 4x + 5$ is one-to-one.

Answer:

$f(x) = x^2 - 4x + 5 = (x^2 - 4x + 4) - 4 + 5 = (x - 2)^2 + 1$. Thus, $f(0) = 5 = f(4)$, so f is not one-to-one. [Or use the Horizontal Line Test.]

101. Determine whether or not the function $g(x)=|x+1|$ is one to one.

Answer:

$g(x)=|x+1|$. Since every number and its negative have the same absolute value, e.g., $|-2| = 2 = |2|$, g is not a one-to-one function.

102. Find the inverse function of $f(x)=\sqrt{3x-1}$ and then verify that f^{-1} and f satisfy the equations:

$f^{-1}(f(x)) = x$ for every x in A and $f(f^{-1}(x)) = x$ for every x in B.

Answer:

$f(x) = \sqrt{3x-1}.\ y = \sqrt{3x-1} \Leftrightarrow 3x - 1 = y^2 \Leftrightarrow x = \frac{1}{3}(y^2 + 1)$. So the inverse function is $f^{-1}(x) = \frac{1}{3}(x^2 + 1)$.

$f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}\left(x^2 + 1\right)\right) = \sqrt{3 \cdot \frac{1}{3}(x^2 + 1) - 1} = \sqrt{x^2 + 1 - 1} = x.$ $f^{-1}(f(x)) = f^{-1}(\sqrt{3x-1}) = \frac{1}{3}\left[(\sqrt{3x-1})^2 + 1\right] =$

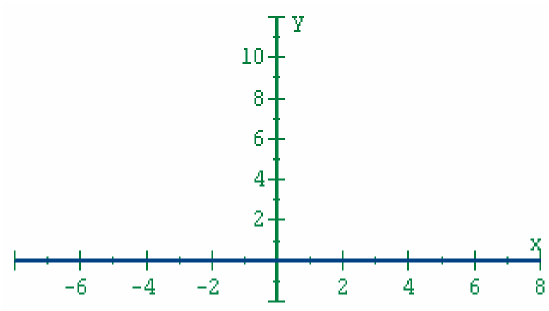
$x - \frac{1}{3} + \frac{1}{3} = x.$

103. For the function *f*(*x*) = $6 - x$:

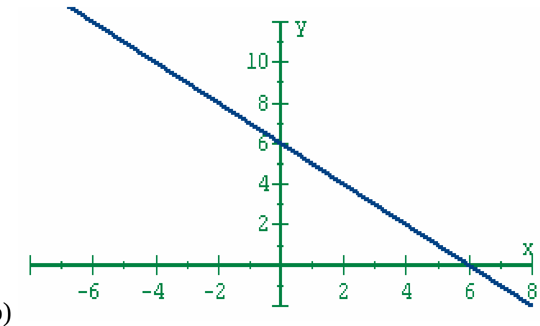
(a) sketch the graph of *f*

(b) use the graph of *f* to sketch the graph of f^{-1}

(c) find f^{-1}.



Answer: (a),(b)



(c) $f(x) = 6 - x$. $y = 6 - x \Leftrightarrow x = 6 - y \quad x = 6 - y$.

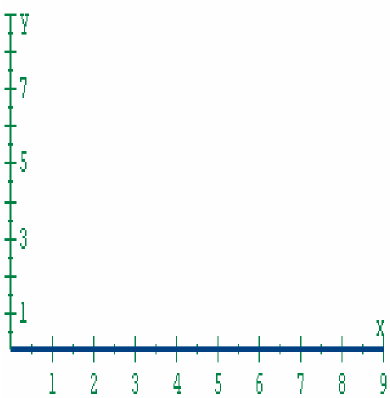
So $f^{-1}(x) = 6 - x$.

104. For the function $f(x) = 9 - x^2, 0 \le x \le 3$:

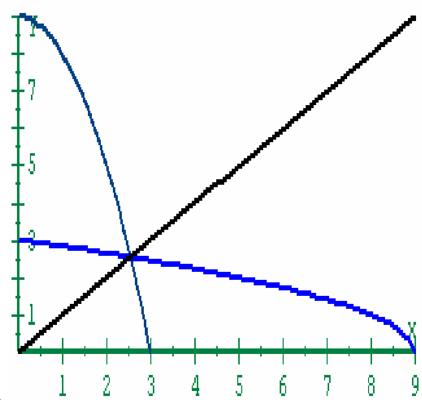
(a)sketch the graph of *f*

(b) use the graph of *f* to sketch the graph of f^{-1}

(c) find an equation for f^{-1}



Answer: (a),(b)

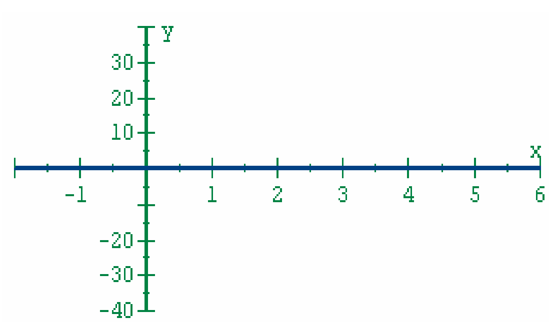


(c) $f(x) = 9 - x^2, 0 \le x \le 3$. $y = 9 - x^2 \Leftrightarrow$

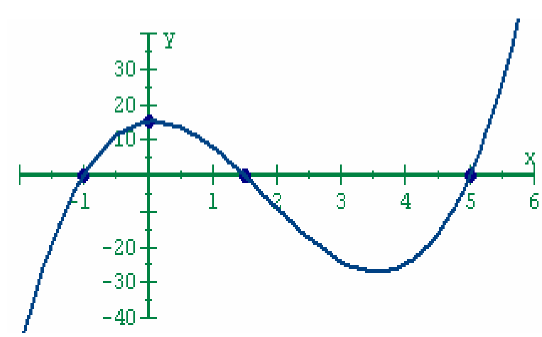
$x^2 = 9 - y \Leftrightarrow x = \sqrt{9 - y}$. So the inverse

function is $f^{-1}(x) = \sqrt{9 - x}$, $0 \le x \le 9$

105. Sketch the graph of the function $y = (2x - 3)(x - 5)(x + 1)$ by first plotting all *x*-intercepts, the *y*-intercept, and sufficiently many other points to detect the shape of the curve, and then filling in the rest of the graph.

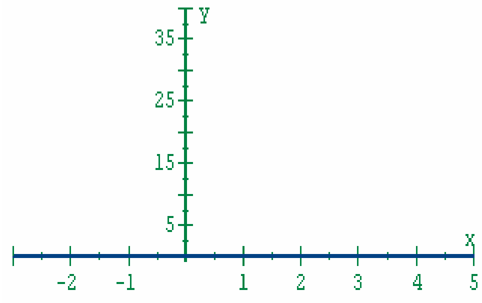


Answer:

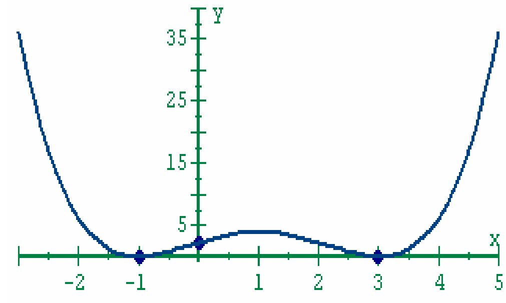


$y = (2x - 3)(x - 5)(x + 1)$

106. Sketch the graph of the function $y = \frac{1}{4}(x+1)^2(x-3)^2$ by first plotting all x-intercepts, the y-intercept, and sufficiently many other points to detect the shape of the curve, and then filling in the rest of the graph.

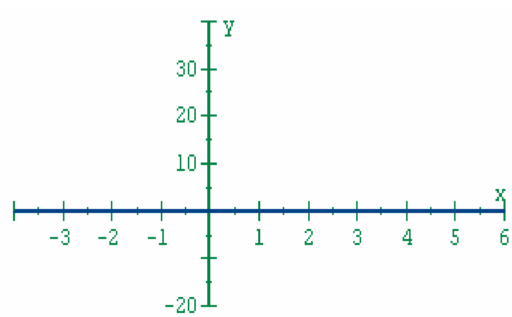


Answer:



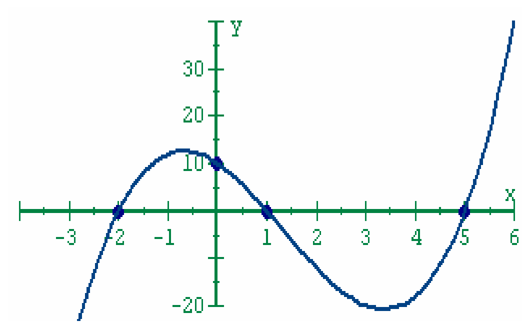
$y = \frac{1}{4}(x+1)^2(x-3)^2$

107. Sketch the graph of the function $y = x^3 - 4x^2 - 7x + 10$ by first plotting all x-intercepts, the y-intercept, and sufficiently many other points to detect the shape of the curve, and then filling in the rest of the graph.



Answer:

$y = x^3 - 4x^2 - 7x + 10$



108. For $\dfrac{2x^3 - x^2 - 5}{x - 3/2}$ find the quotient and remainder.

(a) $Q(x) = 2x^2 - 2x + 1,$ and $R(x) = -1$

(b) $Q(x) = 3x^2 + x + 4,$ and $R(x) = 0$

(c) $Q(x) = 2x^2 + x + 6,$ and $R(x) = 2$

(d) $Q(x) = 2x^2 + 2x + 3,$ and $R(x) = -\frac{1}{2}$

(e) $Q(x) = 2x^2 + 3x - 4,$ and $R(x) = -1$

Answer: (d)

$$\begin{array}{c|cccc} \frac{3}{2} & 2 & -1 & 0 & -5 \\ & & 3 & 3 & \frac{9}{2} \\ \hline & 2 & 2 & 3 & -\frac{1}{2} \end{array}$$

Therefore, $Q(x) = 2x^2 + 2x + 3,$ and $R(x) = -\frac{1}{2}.$

109. Find the value $P(-3)$ of the polynomial $P(x) = x^4 + 4x^3 + 7x^2 + 10x + 15$ using the Remainder Theorem.

(a) 6 (b) 13(c) 15 (d) 21 (e) 33

Answer: (d)

$P(x) = x^4 + 4x^3 + 7x^2 + 10x + 15;\ P(-3)$

-3	1	4	7	10	15
		−3	−3	−12	6
	1	1	4	-2	21

Therefore, $P(-3) = 21$.

110. Use the Factor Theorem to show that $x + 4$ is a factor of the polynomial $P(x) = x^5 + 4x^4 - 7x^3 - 23x^2 + 23x + 12$.

Answer:
$x + 4$ is a factor of $P(x) = x^5 + 4x^4 - 7x^3 - 23x^2 + 23x + 12$ if and only if $P(-4) = 0$

–4	1	4	–7	–23	23	12
		-7	0	28	-20	-12
	1	0	–7	5	3	0

Since $P(-4) = 0$, $x + 4$ is a factor of the polynomial.

111. Find a polynomial of degree 3 with constant coefficient 12 that has zeros $-\frac{1}{2}$, 2, and 3.

Answer:

Since the zeroes are $-\frac{1}{2}$, 2, and 3, a factorization is

$$P(x) = C\left(x + \frac{1}{2}\right)(x-2)(x-3) = \frac{1}{2}C(2x+1)\left(x^2 - 5x + 6\right) = \frac{1}{2}C\left(2x^3 - 10x^2 + 12x + x^2 - 5x + 6\right)$$

$$= \frac{1}{2}C\left(2x^3 - 9x^2 + 7x + 6\right)$$

Since the constant coefficient is 12, $C = 4$, and so the polynomial is $P(x) = 4x^3 - 18x^2 + 14x + 12$.

112. Does there exist a polynomial of degree 4 with integer coefficients that has zeros i, $2i$, $3i$, and $4i$? If so, find it. If not , explain why not.

Answer:
No, there is no polynomial of degree 4 with integer coefficients that has zeros i, $2i$, $3i$, $4i$, since the imaginary roots of polynomial equations with real coefficients come in complex conjugate pairs.

113. If we divide the polynomial $P(x) = x^4 + kx^2 - kx + 2$ by $x + 2$, the remainder is 72. What must the value of k be?

Answer:

Since division of $P(x) = x^4 + kx^2 - kx + 2$ *by* $x + 2$ leaves a remainder of 72, it follows that $P(-2) = 72$. Now,

$P(-2) = (-2)^4 + k(-2)^2 - k(-2) + 2 = 16 + 4k + 2k + 2 = 18 + 6k = 72 \Leftrightarrow 6k = 54 \Leftrightarrow k = 9.$

114. For $P(x) = 6x^4 - x^3 + x^2 - 24$ list all possible rational zeros given by the Rational Roots Theorem, but do not check to see which values are actually roots.

(a) $\pm 1, \pm 2, \pm 4, \pm 6, \pm 8, \pm 12, \pm 18, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{1}{6}$, and $\pm \frac{1}{8}$

(b) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$, and $\pm \frac{1}{6}$

(c) $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 12, \pm 15, \pm 24$, and $\pm \frac{1}{6}$

(d) $\pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{2}{3}, \pm \frac{8}{3}$, and $\pm \frac{1}{6}$

(e) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 9, \pm 12, \pm 18, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}$ and $\pm \frac{1}{6}$

Answer: (b)
$P(x) = 6x^4 - x^3 + x^2 - 24$ has possible rational zeros

115. Find all rational roots of the equation $x^3 - x^2 - 8x + 12 = 0$, and then find the irrational roots, if any.

(a) -1, -2 and 3 (b) -3 *and* $\sqrt{3}$ (c) -3 and 2 (d) -3 *and* $\sqrt{2}$ (e) 2 *and* $\sqrt{3}$

Answer: (c)
$x^3 - x^2 - 8x + 12 = 0$. The possible rational roots are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$. P(x) has 2 variations in sign and hence 0 or 2 positive real roots. P(-x) has 1 variation in sign and hence 1 negative root.

Thus, $(x - 2)(x^2 + x - 6) = 0 \Leftrightarrow (x - 2)(x + 3)(x - 2) = 0$, and so the roots are –3 and 2.

116. Find all rational roots of the equation $x^4 - x^3 - 23x^2 - 3x + 90 = 0$, and then find the irrational roots, if any.

(a) -2, $\sqrt{3}$ *and* $\sqrt{5}$ (b) $-1, \sqrt{2}$ *and* 5 (c) -3, 2 and 5 (d) -2 and 5 (e) -3 *and* $\sqrt{2}$

Answer: (c)
$x^4 - x^3 - 23x^2 - 3x + 90 = 0$. The possible rational roots are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 9, \pm 10, \pm 15, \pm 18, \pm 30, \pm 45, \pm 90$.
Since P(x) has 2 variations in sign, P(x) has 0 or 2 positive real roots. Since P(-x) has 2 variations in sign, P(x) has 0 or 2 negative roots.

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -23 & -3 & 90 \\ & & 2 & 2 & -42 & -90 \\ \hline & 1 & 1 & -21 & -45 & 0 \end{array}$$

$\Rightarrow$ x = 2 is a root, and so $(x-2)(x^3 + x^2 - 21x - 45) = 0$

5
$$\begin{array}{r|rrrr} 5 & 1 & 1 & -21 & -45 \\ & & 5 & 30 & 45 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$\Rightarrow$ x = 5 is a root, and so $(x - 2)(x - 5)(x^2 + 6x + 9) = 0 \Leftrightarrow (x - 2)(x - 5)(x + 3)^2 = 0$. Therefore, the roots are -3, 2, and 5.

117. Use Descartes' Rule of Signs to determine how many positive and negative real zeros the polynomial $3x^5 - 4x^4 + 8x^3 - 5$ can have, and then determine the possible total number of real zeros.

Answer:
$P(x) = 3x^5 - 4x^4 + 8x^3 - 5$. Since P(x) has 3 variations in sign, P(x) can have 3 or 1 positive real zeros. Since $P(-x) = -3x^5 - 4x^4 - 8x^3 - 5$ has 0 variations in sign, P(x) has 0 negative real zeros. Thus, P(x) has 1 or 3 real zeros.

118. Use Descartes' Rule of Signs to determine how many positive and negative real zeros the polynomial $x^4 + x^3 + x^2 + x + 22$ can have, and then determine the possible total number of real zeros.

Answer:
$P(x) = x^4 + x^3 + x^2 + x + 22$. Since P(x) has 0 variations in sign, P(x) has 0 positive real zeros. Since $P(-x) = x^4 - x^3 + x^2 - x + 22$ has 4 variations in sign, P(x) has 4, 2, or 0 negative real zeros. Therefore, P(x) has 0, 2, or 4 real zeros.

119. Show that the given values for *a* and *b* are lower and upper bounds, respectively, for the real roots of the equation.
$3x^4 - 17x^3 + 24x^2 - 9x + 1 = 0$; a = 0, b = 6.

Answer:
$3x^4 - 17x^3 + 24x^2 - 9x + 1 = 0$; a = 0, b = 6. Since P(-x) = $3x^4 + 17x^3 + 24x^2 + 9x + 1$ has 0 variations in sign, P has 0 negative real zeros, and so by Descartes' Rule of Signs, a = 0 is a lower bound.

$$\begin{array}{r|rrrrr} 0 & 3 & -17 & 24 & -9 & 1 \\ & & 0 & 0 & 0 & 0 \\ \hline & 3 & -17 & 24 & -9 & 1 \end{array}$$

$\Rightarrow$ alternating signs, therefore, a=0 is lower bound

$$\begin{array}{r|rrrrr} 6 & 3 & -17 & 24 & -9 & 1 \\ & & 18 & 6 & 180 & 1026 \\ \hline & 3 & 1 & 30 & 171 & 1027 \end{array}$$

$\Rightarrow$ all positive. Therefore, b = 6 is an upper bound

120. Find integers that are upper and lower bounds for the real roots of the equation $x^5 - x^4 + 1 = 0$

Answer:

$x^5 - x^4 + 1 = 0$

$$\begin{array}{r|rrrrrr} 1 & 1 & -1 & 0 & 0 & 0 & 1 \\ & & 1 & 0 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 0 & 0 & 0 \\ & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & 1 \end{array}$$

$\Rightarrow$ all non-negative, so 1 is an upper bound
$\Rightarrow$ Alternating positive and negative
$\Rightarrow$ Therefore, a lower bound is –1 and an upper is 1.

121. Find all rational roots of the equation $4x^4 - 25x^2 + 36 = 0$, and then find the irrational roots, if any.

(a) ± 1 and ± 2 (b) ± 2 and $\pm \frac{3}{2}$ (c) ± 1 and $\sqrt{2}$ (d) ± 2 and $\sqrt{3}$ (e) ± 1 and $1 \pm \sqrt{2}$

Answer: (b)

$4x^4 - 25x^2 + 36 = 0$ has possible rational roots $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{9}{2}, \pm \frac{9}{4}$. Since P (x) has 2 variations in sign, there are 0 or 2 positive real roots. Since P(-x) = $4x^4 - 25x^2 + 36$ has 2 variations in the sign, there are 0 or 2 negative real roots.

2	4	0	-25	0	36
		8	16	-18	-36
	4	8	-9	-18	0

3/2	4	8	-9	-18
		6	21	-18
	4	14	12	0

$\Rightarrow$ x = 2 is the root, and so $(x-2)(4x^3 + 8x^2 - 9x - 18) = 0$

$\Rightarrow$ all positive

$\Rightarrow$ $x = \frac{3}{2}$ is a root, and so $(x-2)(2x-3)(2x^2 + 7x + 6) = 0 \Leftrightarrow (x-2)(2x-3)(2x+3)(x+2) = 0$ Therefore, the roots are $\pm 2, \pm \frac{3}{2}$.

122. Find the x- and y- intercepts of the function $y = \frac{2}{x^3 - 2x + 8}$

(a) x-intercepts $\frac{1}{2}$; y-intercepts $\frac{1}{3}$ (b) No x- intercepts; y- intercepts $\frac{1}{3}$

(c) No x- intercepts; y- intercepts $\frac{1}{4}$ (d) x- intercepts $\pm \frac{1}{4}$; y- intercepts $\frac{1}{4}$ (e) x- intercepts $\frac{1}{2}$; y – intercepts $\pm \frac{1}{4}$

Answer: (c)

$y = \frac{2}{x^3 - 2x + 8}$. When x = 0, $y = \frac{2}{0 - 0 + 8} = \frac{1}{4}$, and so the y- intercepts $\frac{1}{4}$. Since it is impossible for y to equal 0, there is no x- intercepts.

123. Find the x- and y- intercepts of the function $y = \frac{x^2 + 12}{3x}$.

(a) No x- intercept; y- intercept $\frac{1}{4}$ (b) No x- intercept; y- intercepts $\pm \frac{1}{3}$ (c) x-intercept $\frac{2}{3}$; no y- intercept

(d) No intercept (e) x- intercept $\frac{2}{3}$; y- intercepts $\pm \frac{1}{3}$

Answer: (d)

$y = \frac{x^2 + 12}{3x}$. When x = 0, $\frac{0 + 12}{0}$ which is undefined, and so there is no y- intercept. Since $x^2 + 12 \rangle 0$ for all x, it is impossible for y to equal 0, so there is no x- intercept.

124. Find all asymptotes (including vertical, horizontal) of the function y = $\frac{3x+1}{x-3}$.

(a) No horizontal; vertical: x = 3 (b) Horizontal: y = $\frac{1}{3}$; vertical: x = 3

(c) Horizontal: y = $\frac{1}{3}$; vertical: x = $\frac{1}{3}$ (d) Horizontal: y = 3; vertical: x = ± 3

(e) Horizontal: y = 3; vertical: x = 3

Answer: (e)

y= $\frac{3x+1}{x-3} = \frac{3+1/x}{1-3/x} \to 3$ as x $\to \infty$. The horizontal asymptote is y = 3. There is a vertical asymptote when x-3 = 0 $\Leftrightarrow$ x = 3, and so the vertical asymptote is x = 3.

125. Find all asymptotes (including vertical, horizontal) of the function y = $\frac{2x-5}{x^2+x+1}$.

(a) no asymptote (b) Horizontal: y = 0; vertical: x = $\frac{5}{2}$ (c) Horizontal: y = 2; vertical: x = $\frac{5}{2}$

(d) Horizontal: y = 0; no vertical (e) Horizontal: y = $\pm$ 1; vertical: x = -1

Answer: (d)

y = $\frac{2x-5}{x^2+x+1}$. The vertical asymptotes occur when $x^2 + x + 1 = 0$ $\Leftrightarrow$ x = $\frac{-1\pm\sqrt{1-4\cdot1\cdot1}}{2}$ = –

$\frac{-1\pm\sqrt{-3}}{2}$ which is imaginary, and so there are no vertical asymptotes. Since y = $\frac{2x-5}{x^2+x+1}$ =

$\frac{2/x-5/x^2}{1+1/x+1/x^2} \to 0$ as x $\to \infty$, y = 0 is the horizontal asymptote.

126. Find all asymptotes (including vertical, horizontal) of function y = $\frac{6x^4+x^2-1}{x^2+64}$.

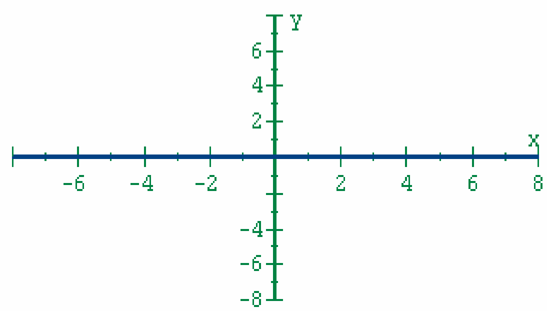
(a) Horizontal: y = 4; vertical: x = - 4 (b) Horizontal: y = - 4; vertical: x = 2

(c) No horizontal; vertical: x = $\sqrt{3}$ (d) No horizontal; no vertical; slant: y = 2x (e) No asymptote

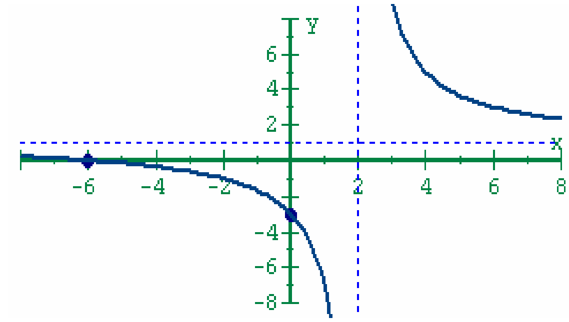
Answer: (e)

y = $\frac{6x^4+x^2-1}{x^2+64}$. Since $x^2 + 64 \geq 64$ for all x, there are no vertical asymptote. There are also no horizontal asymptotes.

127. Find the intercepts and asymptotes, and then graph the rational function y = $\frac{x+6}{x-2}$



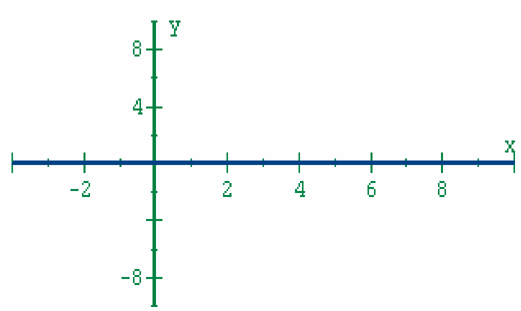
Answer:



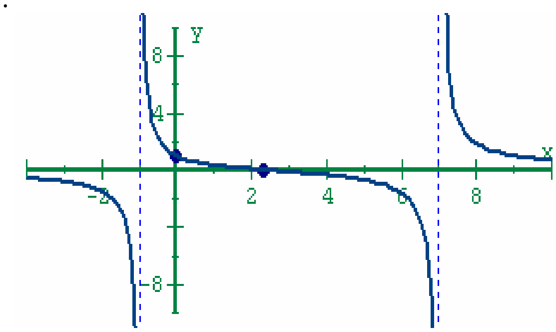
= $\frac{x+6}{x-2}$. When x = 0, y = $\frac{0+6}{0-2}$ = - 3, and so the y = intercept is –3. When y = 0, x + 6 = 0 $\Leftrightarrow$ x = -6, and so the x- intercept is –6.

Since y = $\frac{x+6}{x-2} = \frac{1+6/x}{1-2/x} \rightarrow 1$ as x $\rightarrow \infty$, y = 1 is the horizontal asymptote. x- 2= 0 $\Leftrightarrow$ x = 2 is the vertical asymptote.

128. Find the intercepts and asymptotes, and then graph the rational function y = $\frac{3x-7}{x^2-6x-7}$.



Answer:

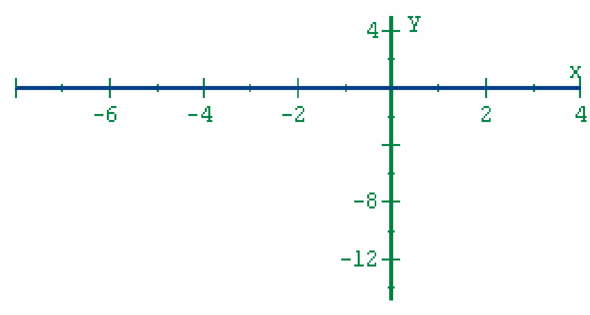


y = $\frac{3x-7}{x^2-6x-7} = \frac{3x-7}{(x-7)(x+1)}$. When x = 0,

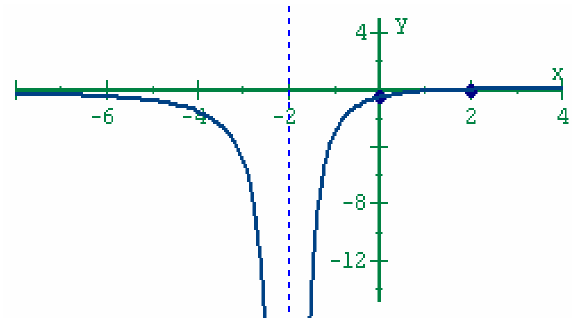
y = $\frac{0-7}{0-0-7}$ = 1, and so the y-intercept is 1. When y = 0,

3x – 7 = 0 $\Leftrightarrow x = \frac{7}{3}$, and so the x-intercept is $\frac{7}{3}$. The horizontal asymptote is y = 0 and the vertical asymptotes are x = -1 and x = 7

129. Find the intercepts and asymptotes, and then graph the rational function y = $\dfrac{x-2}{(x+2)^2}$.

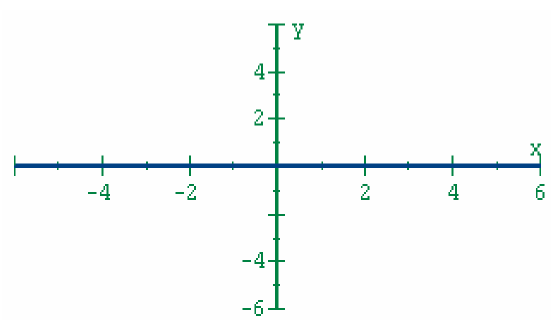


Answer:

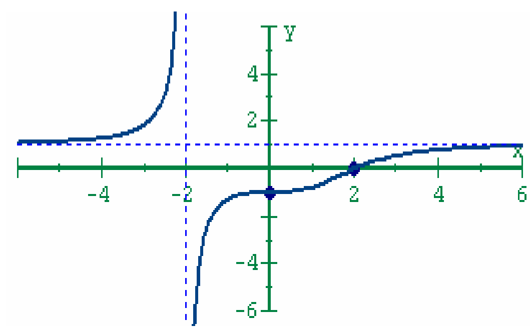


y = $\dfrac{x-2}{(x+2)^2}$. When x = 0, y = $\dfrac{0-2}{(0+2)^2} = -\dfrac{1}{2}$, and so the y-intercept is $-\dfrac{1}{2}$. When y = 0, x – 2 = 0, and so the x-intercept is 2. The horizontal asymptote is y = 0 and the vertical asymptote is x + 2 = 0 $\Leftrightarrow$ x = -2. (Note that the graph approaches y = 0 from above as $x \to \infty$.)

130. Find the intercepts and asymptotes, and then graph the rational function y = $\dfrac{x^3-8}{x^3+8}$.

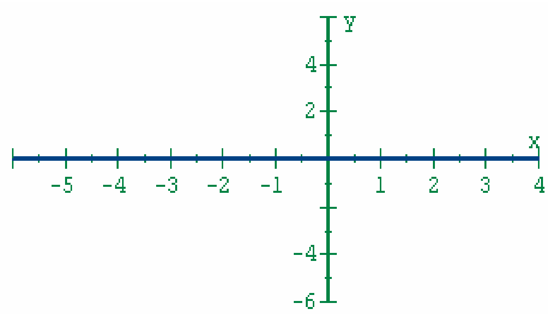


Answer:

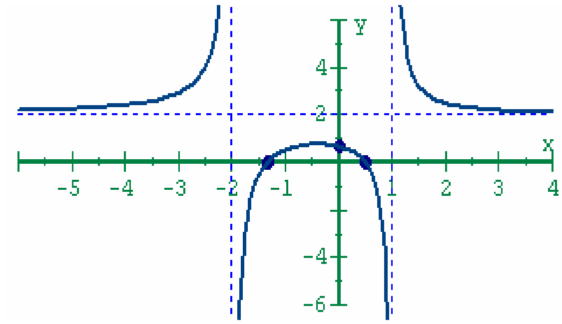


y = $\dfrac{x^3-8}{x^3+8} = \dfrac{1-8/x^3}{1+8/x^3} \to 1$ as x $\to \infty$. When x = 0, y = $\dfrac{0-8}{0+8} = -1$, and so the y-intercept is -1. When y = 0, $x^3 - 8 = 0 \Leftrightarrow$ x = 2, and so the x-intercept is 2. The horizontal asymptote is y = 1 and the vertical asymptote is $x^3 + 8 = 0 \Leftrightarrow$ x = -2.

131. Find the intercepts and asymptotes, and then graph the rational function y = $\dfrac{6x^2+5x-4}{3x^2+3x-6}$.

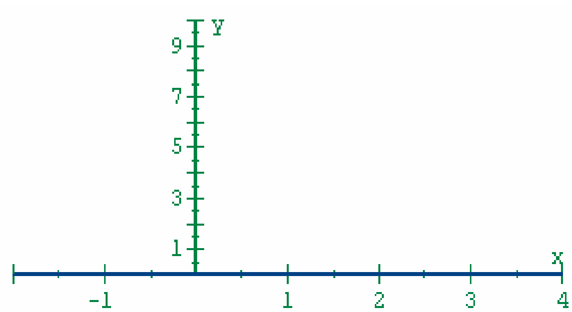


Answer:

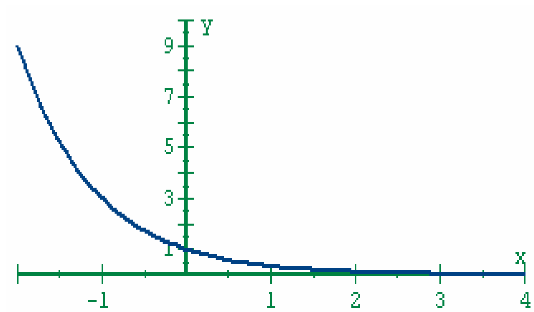


y = $\dfrac{6x^2+5x-4}{3x^2+3x-6} = \dfrac{(3x+4)(2x-1)}{3(x-1)(x+2)} \to 2$ as x $\to \infty$. When x = 0, y = $\dfrac{(0+4)(0-1)}{3(0-1)(0+2)} = \dfrac{2}{3}$, and so the y-intercept is $\dfrac{2}{3}$. When y = 0, 3x + 4 = 0 $\Leftrightarrow$ x = $-\dfrac{4}{3}$ or 2x – 1 = 0 $\Leftrightarrow x = \dfrac{1}{2}$, and so the x-intercepts are $-\dfrac{4}{3}$ and $\dfrac{1}{2}$ The horizontal asymptote is y = 2 and the vertical asymptotes are x + 2 = 0 $\Leftrightarrow$ x = -2 and x-1=0 $\Leftrightarrow$ x=1.

132. Graph the function f (x) = 3^{-x}, not by plotting points but by applying your knowledge of the general shape of graphs of the form a ^{x}. State the domain, range , and asymptote of the function.



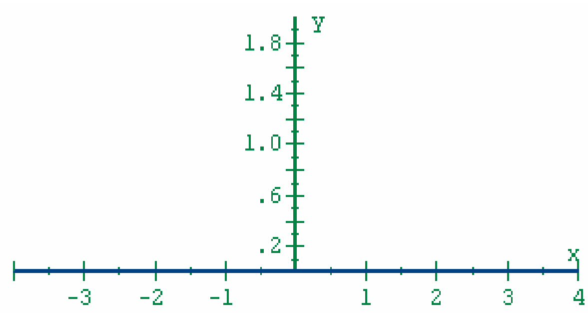
Answer:

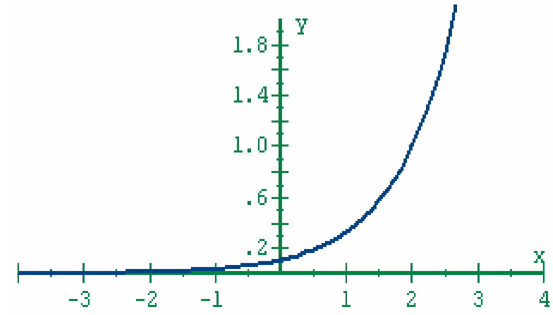


f(x) = 3^{-x}. D: ($-\infty$, ∞), R: (0, ∞), A: y = 0

133. Graph the function $g(x) = 3^{x-2}$, not by plotting points but by applying your knowledge of the general shape of graphs of the form a^x. State the domain, range, and asymptote of the function.

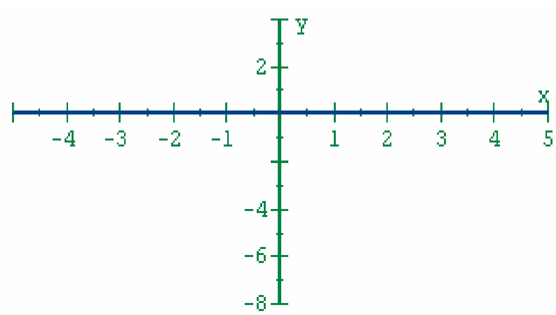
Answer: $D: (-\infty,\ \infty)\ \ R: (0,\ \infty)\ \ A: y = 0$

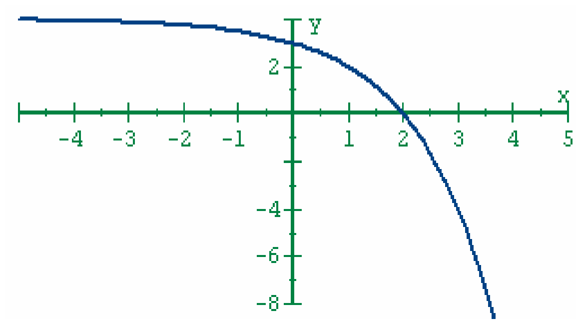




134. Graph the function $g(x) = 4 - 2^x$, not by plotting points but by applying your knowledge of the general shape of graphs of the form a^x. State the domain, range, and asymptote of the function.

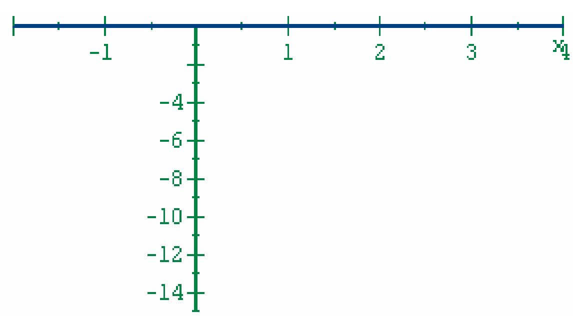
Answer: $D:(-\infty,\infty)\quad R:(-\infty, 4)\quad A: y = 4$

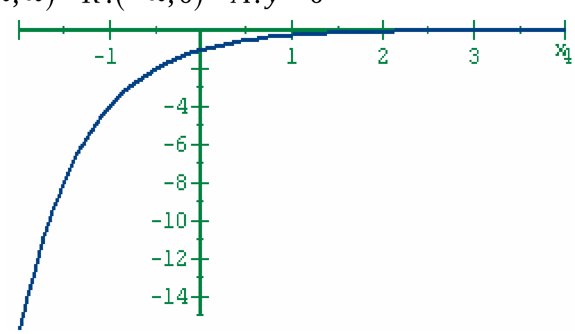




135. Graph the function $y = -\left(\frac{1}{4}\right)^x$, not by plotting points but by applying your knowledge of the general shape of graphs of the form a^x. State the domain, range, and asymptote of the function.

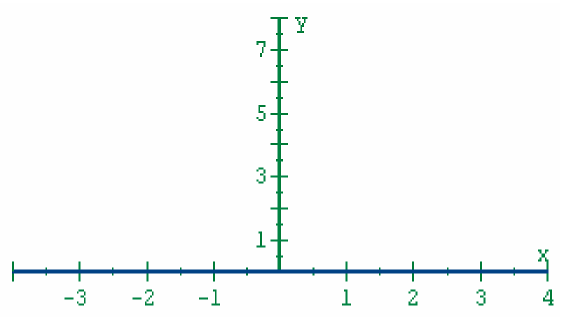
Answer: $D:(-\infty,\infty)\quad R:(-\infty,0)\quad A: y = 0$

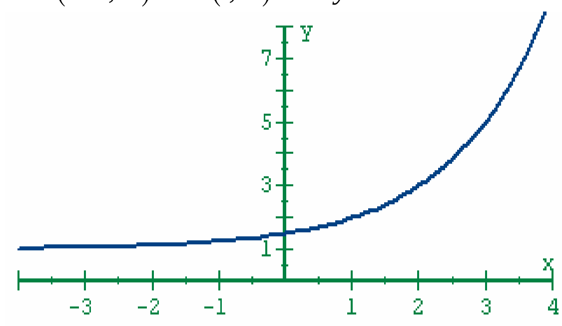




136. Graph the function $y = 1 + 2^{x-1}$, not by plotting points but by applying your knowledge of the general shape of graphs of the form a^x. State the domain, range, and asymptote of the function.

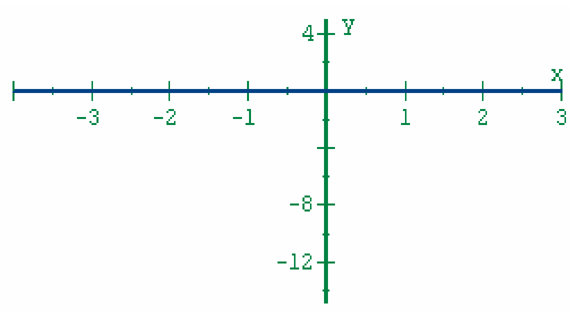
Answer: $D:(-\infty,\infty)$ $R:(1,\infty)$ $A: y=1$

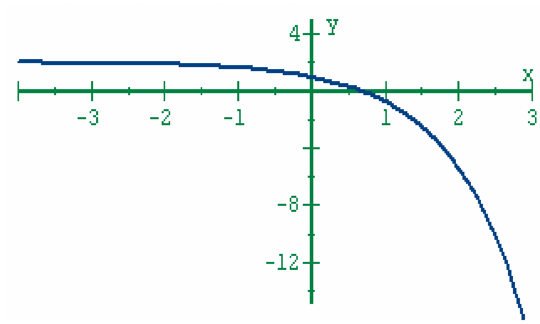




137. Graph the function $y = 2 - e^x$, not by plotting points but by starting with the graph of $y = e^x$. State the domain, range, and asymptote of the function.

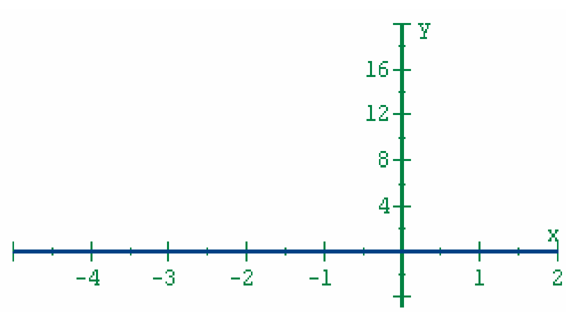
Answer: $D:(-\infty,\infty)$ $R:(-\infty,2)$ $A: y=2$

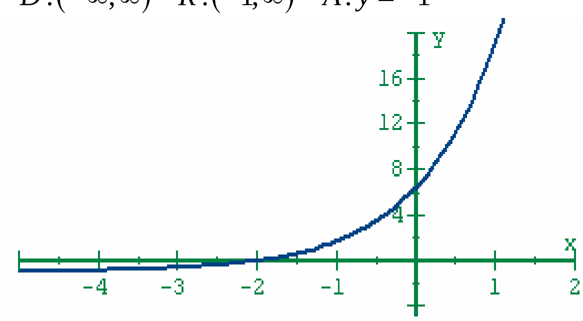




138. Graph the function $y = e^{x+2} - 1$, not by plotting points but by starting with the graph of $y = e^x$. State the domain, range, and asymptote of the function.

Answer: $D:(-\infty,\infty)$ $R:(-1,\infty)$ $A: y=-1$





139. Express the equation $\log_6 1 = 0$ in exponential form.

(a) $6^1 = 0$ (b) $0^6 = 1$ (c) $6^0 = 1$ (d) $1^6 = 1$ (e) $6^1 = 6$

Answer: (c)
$\log_6 1 = 0 \Leftrightarrow 6^0 = 1$

140. Express the equation $\log_{27} 9 = \frac{2}{3}$ in exponential form.

(a) $3^3 = 27$ (b) 6/9= 2/3 (c) $9 = 3^2$ (d) $27^{2/3} = 9$ (e) $9^{3/2} = 27$

Answer: (d)

$\log_{27} 9 = \frac{2}{3} \Leftrightarrow 27^{\frac{2}{3}} = 9$

141. Express the equation $\log_2 (\frac{1}{8}) = -3$ in exponential form.

(a) $2^3 = 8$ (b) $2^{-3} = \frac{1}{8}$ (c) $8^{-1/8} = 2$ (d) $27^{2/3} = 9$ (e) $-3^8 = 2$

Answer: (b)

$\log_2(\frac{1}{8}) = -3 \Leftrightarrow 2^{-3} = \frac{1}{8}$

142. Express the equation $\log_r v = w$ in exponential form.

(a) $v^w = r$ (b) $r^w = v$ (c) $v^r = w$ (d) $w^r = v$ (e) $w^v = r$

Answer: (b)

$\log_r v = w \Leftrightarrow r^w = v$

143. Express the equation $10^5 = 100{,}000$ in logarithmic form.

(a) $\log_{10} 10{,}000 = 4$ (b) $\log_{10} 100{,}000 = 10$ (c) $\log_{10} 10 = 5$
(d) $\log_{100{,}000} 5 = 10$ (e) $\log_{10} 100{,}000 = 5$

Answer: (e)

$10^5 = 100{,}000 \Leftrightarrow \log_{10} 100{,}000 = 5$

144. Express the equation $16^{1/2} = 4$ in logarithmic form.

(a) $\log_4 \frac{1}{2} = \frac{1}{16}$ (b) $\log_4 16 = 2$ (c) $\log_4 2 = \frac{1}{16}$ (d) $\log_4 2 = 2$ (e) $\log_{16} 4 = \frac{1}{2}$

Answer: (e)

$16^{1/2} = 4 \Leftrightarrow \log_{16} 4 = \frac{1}{2}$

145. Express the equation $5^{-1} = \frac{1}{5}$ in logarithmic form.

(a) $\log_5 (-\frac{1}{5}) = -1$ (b) $\log_5 5 = 1$ (c) $\log_1 \frac{1}{5} = -\frac{1}{5}$ (d) $\log_5 \frac{1}{5} = -1$ (e) $\log_5 1 = -5$

Answer: (d)

$5^{-1} = \frac{1}{5} \Leftrightarrow \log_5 \frac{1}{5} = -1$

146. Express the equation $10^m = n$ in exponential form.

(a) $\log_{10} n = m$ (b) $\log_{10} m = n$ (c) $\log_m 10 = n$ (d) $\log_n m = 10$ (e) $\log_n 10 = m$

Answer: (a)

$10^m = n \Leftrightarrow \log_{10} n = m$

147. Evaluate the expression $\log_2 16$.

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

Answer: (b)
$\log_2 16 = \log_2 2^4 = 4$

148. Evaluate the expression $\log_7 7^{13}$

(a) 7 (b) 11 (c)13 (d) 17 (e) 20

Answer: (c)
$\log_7 7^{13} = 13$

149. Evaluate the expression $\log_5 1$.

(a) -1 (b) 1 (c) 3 (d) 5 (e) 0

Answer: (e)
$\log_5 1 = \log_5 5^0 = 0$

150. Evaluate the expression $\log_5 625$.

(a) 3 (b) 4 (c) 5 (d) 125 (e) 625

Answer: (b)
$\log_5 625 = \log_5 5^4 = 4$

151. Evaluate the expression $3^{\log_3 7}$.

(a) 7 (b) 10 (c) 11 (d) 24 (e) 49

Answer: (a)
$3^{\log_3 7} = 7$

152. Evaluate the expression $\log_8 16$.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$ (e) 2

Answer: (d)
$\log_8 16 = \log_8 8^{4/3} = \frac{4}{3}$

153. Solve the equation $\log_3 x = 4$ for x.

Answer:
$\log_3 x = 4 \Leftrightarrow x = 3^4 = 81$

154. Solve the equation $\log_3(2 - x) = 3$ for x.

Answer:
$\log_3(2 - x) = 3 \Leftrightarrow 2 - x = 27 \Leftrightarrow -x = 25 \Leftrightarrow x=-25$

155. Solve the equation $\log_x 5 = \frac{1}{2}$ for x.

Answer:

$\log_x 5 = \frac{1}{2} \Leftrightarrow 5 = x^{1/2} \Leftrightarrow 25 = x$

156. Use a calculator to evaluate the expression $\ln \sqrt{3}$.

(a) 0.2465 (b) 0.5493 (c) 0.6489 (d) 0.9954 (e) 1.066

Answer: (b)

$\ln \sqrt{3} \approx 0.5493$

157. Use a calculator to evaluate the expression ln 0.5.

(a) -2.2241 (b) -1.1042 (c) -1.0314 (d) -0.9421 (e) -0.6931

Answer: (e)

$\ln 0.5 \approx -0.6931$

158. Use a calculator to evaluate the expression $\ln \pi$.

(a) 0.2465 (b) 1.1211 (c) 1.1447 (d) 1.3043 (e) 2.0104

Answer: (c)

$\ln \pi \approx 1.1447$

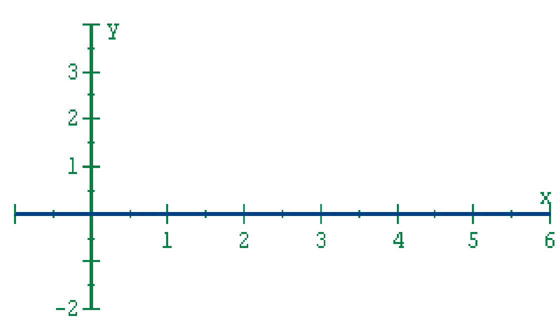
159. Use a calculator to evaluate the expression ln 107.9.

(a) 2.0302 (b) 3.1660 (c) 3.9025 (d) 4.6812 (e) 5.0029

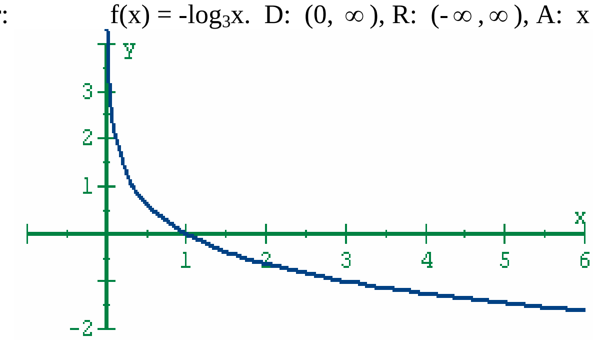
Answer: (d)

$\ln 107.9 \approx 4.6812$

160. Graph the function $f(x) = -\log_3 x$, not by plotting points but by applying your knowledge of the general shape of the logarithmic function. State the domain, range, and asymptote.

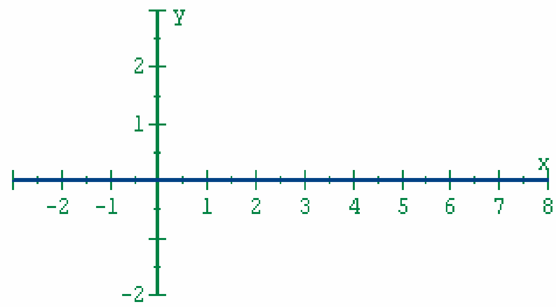


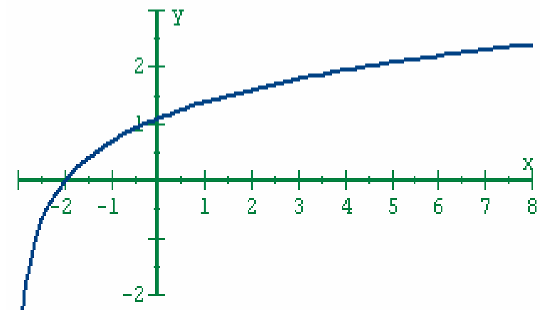
Answer: $f(x) = -\log_3 x$. D: $(0, \infty)$, R: $(-\infty, \infty)$, A: $x = 0$



161. Graph the function $g(x) = \ln(x + 3)$, not by plotting points but by applying your knowledge of the general shape of the logarithmic function. State the domain, range, and asymptote.

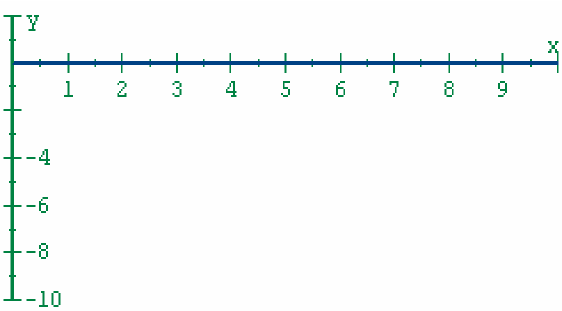
Answer: $g(x) = \ln(x + 3)$. D: $(-3, \infty)$, R: $(-\infty, \infty)$, A: $x = -3$

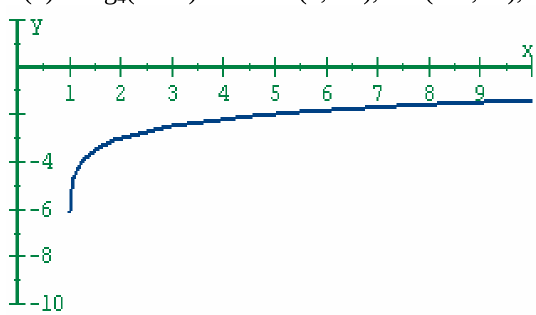




162. Graph the function $y = \log_4(x - 1) - 3$, not by plotting points but by applying your knowledge of the general shape of the logarithmic function. State the domain, range, and asymptote.

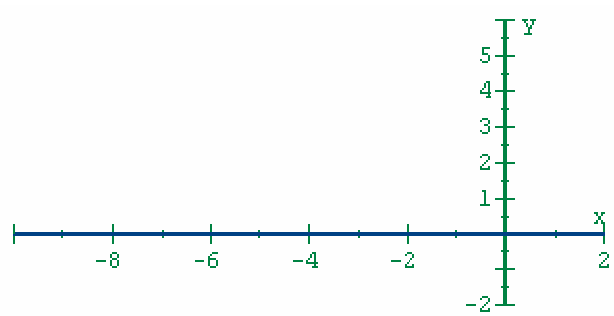
Answer: $f(x) = \log_4(x - 1) - 3$. D: $(1, \infty)$, R: $(-\infty, \infty)$, A: $x = 1$

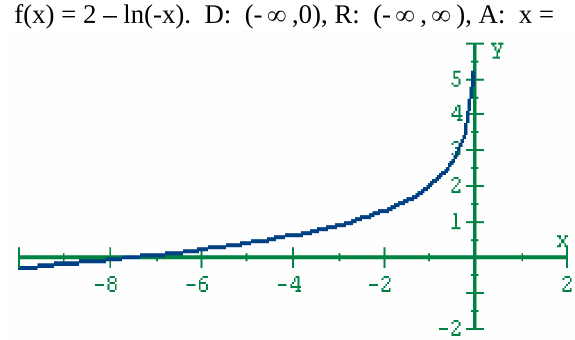




163. Graph the function $y = 2-\ln(-x)$, not by plotting points but by applying your knowledge of the general shape of the logarithmic function. State the domain, range, and asymptote.

Answer: $f(x) = 2 - \ln(-x)$. D: $(-\infty, 0)$, R: $(-\infty, \infty)$, A: x =





164. Find the domain of the function $f(x) = \log_2(10 - 2x)$.

Answer:
$f(x) = \log_2(10 - 2x)$. Then $10 - 2x > 0 \Leftrightarrow x < 5$ and so D is $(-\infty, 5)$

165. Which is larger, $\log_5 26$ or $\log_6 35$?

Answer:
Since $\log_5 x$ is increasing, $\log_5 26 > \log_5 25 = 2$. Also, because $\log_6$ is increasing, $\log_6 35 < \log_6 36 = 2$. Therefore $\log_6 35 < 2 < \log_5 26$ and so $\log_5 26$ is larger.

166. Use the Laws Of Logarithms to rewrite the expressions $\log_3 \left(\frac{x}{4}\right)$ in a form with no logarithms of products, quotients, or powers.

(a) $\log_3 x + \log_3 4$ (b) $\log_3 x - \log_3 4$ (c) $\log_3(x-4)$ (d) $\log_4 x - \log_4 3$ (e) $\frac{\log_3 x}{\log_3 4}$

Answer: (b)

$\log_3 (\frac{x}{4}) = \log_3 x - \log_3 4$

167. Use the Laws of Logarithms to rewrite the expression ln (ex) in a form with no logarithms of products, quotients,or powers.

(a) 2+2 ln x (b) 1- ln x (c) e ln x (d) –ln x (e) 1+ ln x

Answer: (e) ln(ex) = ln e + ln x = 1+ ln x

168. Use the Laws of Logarithms to rewrite the expression $\log_6 \sqrt[5]{13}$ in a form of no logarithms of products, quotients, or powers.

(a) $\frac{1}{6}\log_5 13$ (b) $\log_6 13 - \log_6 5$ (c) $\sqrt[5]{\log_6} 13$ (d) $\frac{1}{5}\log_6 13$ (e) $\frac{1}{3}\log_6 5$

Answer: (d) $\log_6 \sqrt[5]{13} = \frac{1}{5}\log_6 13$

169. Use the Laws of Logarithms to rewrite the expression $\log_3(xy)^7$ in a form with no logarithms of products, quotients, or powers.

(a) $3(\log_7 x + \log_7 y)$ (b) $7(\log_3 x - \log_3 y)$ (c) $3(\log_3 x - \log_3 y)$
(d) $7(\log_3 x + \log_3 y)$ (e) $7\log_3 x + \log_7 y)$

Answer: (d)
$\log_3(xy)^7 = 7[\log_3(xy)] = 7(\log_3 x + \log_3 y)$

170. Use the Laws of Logarithms to rewrite the expression $\log_a \frac{x^3}{y^2 z^2}$ in a form with no logarithms of products, quotients, or powers.

(a) $3 \log_a x - 2(\log_a y + \log_a z)$ (b) $3(\log_a x - \log_a y - \log_a z)$ (c) $\log_a x - \log_a y - \log_a z$
(d) $\frac{3}{2}\log_a x - \frac{2}{3}(\log_a y + \log_a z)$ (e) $3x \log_a y + 3y \log_a z$

Answer: (a) $\log_a \frac{x^3}{y^2 z^2} = \log_a x^3 - \log_a y^2 z^2 = 3\log_a x - 2(\log_a y + \log_a z)$

171. Use the Laws of Logarithms to rewrite the expression $\ln \sqrt[3]{4r s^4}$ in a form with no logarithms of products, quotients, or powers.

(a) $\frac{1}{4}(\ln 3 + \ln r + 3\ln s)$ (b) $\frac{1}{3}(4\ln r + \ln s)$ (c) $\ln 4 - \ln 3(\ln r + 4 \ln s)$
(d) $\frac{1}{3}(\ln 4 + \ln r + 4\ln s)$ (e) $\frac{1}{3}(\ln 4 - \ln r - \frac{4}{3}\ln s)$

Answer: (d) $\ln \sqrt[2]{4r s^4} = \frac{1}{3}\ln(4rs^4) = \frac{1}{3}(\ln 4 + \ln r + 4 \ln s)$

172. Use the Laws of Logarithms to rewrite the expression $\log \frac{a^3}{b\sqrt[3]{c}}$ in a form with no logarithms of products, quotients, or powers.

(a) $\frac{1}{3}$(log a – log b – 3log c) (b) 3 log a – (3log b + log c) (c) 3log a – (log b + $\frac{1}{3}$ log c)

(d) 2log a – logb - $\frac{1}{3}$ logc (e) log a – (log b)(log c)

Answer: (c) $\log \frac{a^3}{b\sqrt[3]{c}} = \log a^3 - \log(b\sqrt[3]{c}) = 3\log a - (\log b + \frac{1}{3}\log c)$

173. Use the Laws of Logarithms to rewrite the expression $\log_4 \sqrt{\frac{x+1}{x-1}}$ in a form with no logarithms of products, quotients, or powers.

(a) $\frac{1}{2}[\log_4(x-1) + \log_4(x+1)]$ (b) $\frac{1}{4}[\log_3(x+2) - \log_3(x-2)]$ (c) $\frac{1}{2}[\log_4(x+1) + \log_4(x-1)]$

(d) $\log_2(x+1) - \log_2(x-1)$ (e) $\frac{1}{2}[\log_4(x+1) - \log_4(x-1)]$

Answer: (e)

$$\log_4 \sqrt{\frac{x+1}{x-1}} = \frac{1}{2}\log_4\left(\frac{x+1}{x-1}\right) = \frac{1}{2}[\log_4(x+1) - \log_4(x-1)]$$

174. Use the Laws of Logarithms to rewrite the expression $\ln \frac{4x^3}{(x-1)^7}$ in a form with no logarithms of products, quotients, or powers.

(a) ln 4 + 3ln x – 7ln(x – 1) (b) ln 12 + ln x – ln 7 + ln(x –1) (c) 3ln 4 + ln x – ln 7 – ln x
(d) ln 4 – ln 3 + ln x – ln(x – 1) (e) 2 + xln 3 – ln 7

Answer: (a)

$$\ln \frac{4x^3}{(x-1)^7} = \ln(4x^3) - \ln[(x-1)^7] = \ln 4 + 3\ln x - 7\ln(x-1)$$

175. Use the Laws of Logarithms to rewrite the expression $\log \frac{1}{\sqrt[3]{1+x}}$ in a form with no logarithms of products, quotients, or powers.

(a) $\frac{1}{3}$log(-1-x) (b) 1 - $\frac{1}{3}$log x (c) -3log x (d) -$\frac{1}{3}$log(1 + x) (e) 1 + $\frac{1}{3}$log(x – 1)

Answer: (d)

$$\log \frac{1}{\sqrt[3]{1+x}} = \log 1 - \log \sqrt[3]{1+x} = 0 - \frac{1}{3}\log(1+x) = -\frac{1}{3}\log(1+x)$$

176. Use the Laws of Logarithms to rewrite the expression $\log \frac{10^{2x}}{x(x^2-1)(x^3-2)}$ in a form with no logarithms of products, quotients, or powers.

(a) $2x - [\log x + \log(x^2 - 1) + \log(x^3 - 2)]$
(b) $20 - (6\log x - 3)$
(c) $20x - [\log x + 2\log(x - 1) + 3\log(x - 2)]$
(d) $2x - [\log x + \log(x + 1) + 2\log(x - 1)]$
(e) $2x - [3\log x + \log(x^3) - 2]$

Answer: (a)

$$\log \frac{10^{2x}}{x(x^2-1)(x^3-2)} = \log 10^{2x} - \log(x(x^2 - 1)(x^3 - 2)) = 2x - [\log x + \log(x^2 - 1) + \log(x^3 - 2)]$$

177. Evaluate the expression $\log_2 144 - \log_2 9$.

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

Answer: (b)

$$\log_2 144 - \log_2 9 = \log_2 \frac{144}{9} = \log_2 16 = \log_2 2^4 = 4$$

178. Evaluate the expression $\log \sqrt[3]{0.001}$.

(a) -1 (b) 2 (c) -3 (d) $-\frac{1}{2}$ (e) $\frac{1}{3}$

Answer: (a)

$$\log \sqrt[3]{0.001} = \frac{1}{3}\log(10^{-3}) = -1$$

179. Evaluate the expression $\log_6 12 + \log_6 18$.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Answer: (c)

$$\log_6 12 + \log_6 18 = \log_6(12 \cdot 18) = \log_6 6^3 = 3$$

180. Rewrite the expression $\log 8 + \frac{1}{3}\log 9 - \log 2$ as a single logarithm.

Answer:

$$\log 8 + \frac{1}{3}\log 9 - \log 2 = \log 8\sqrt[3]{9} - \log 2 = \log \frac{8\sqrt[3]{9}}{2} = \log(4\sqrt[3]{9})$$

181. Rewrite the expression $\log_4(x^2 - 1) - \log_4(x + 1)$ as a single logarithm.

Answer:

$$\log_4(x^2 - 1) - \log_4(x + 1) = \log_4 \frac{x^2-1}{x+1} = \log_4(x - 1)$$

182. Rewrite the expression ln(a – b) – ln(a + b) + 2ln c as a single logarithm.

Answer:

$\ln(a-b) - \ln(a+b) + 2\ln c = \ln\frac{a-b}{a+b} + \ln c^2 = \ln\frac{c^2(a-b)}{a+b}$

183. Rewrite the expression $\frac{1}{3}[\log_4 x + 3\log_4 y - 2\log_4 z]$ as a single logarithm.

Answer:

$\frac{1}{3}[\log_4 x + 3\log_4 y - 2\log_4 z] = \frac{1}{3}\log_4 \frac{xy^3}{z^2} = \log_4 \sqrt[3]{\frac{xy^3}{z^2}}$

184. Use the change of base formula and a calculator to evaluate the logarithm $\log_5 3$ correct to six decimal places.

Answer: $\log_5 3 = \frac{\log 3}{\log 5} \approx 0.682606$

185. Use the change of base formula and a calculator to evaluate the logarithm $\log_5 85$ correct to six decimal places.

Answer:

$\log_5 85 \frac{\log 85}{\log 5} \approx 2.760374$

186. Find the solution of the equation $8^{1-x} = 5$ correct to four decimal places.

(a) 0.2260 (b) 0.2756 (c) 0.3045 (d) 0.9384 (e) 1.7563

Answer: (a)

$8^{1-x} = 5 \Leftrightarrow \log 8^{1-x} = \log 5 \Leftrightarrow (1-x)\log 8 = \log 5 \Leftrightarrow 1 - x = \frac{\log 5}{\log 8} \Leftrightarrow x = 1 - \frac{\log 5}{\log 8} \approx 0.2260$

187. Find the solution of the equation $3^{x/12} = 0.1$ correct to four decimal places.

(a) -32.2566 (b) -29.0743 (c) -25.1508 (d) -19.0828 (e) -7.0523

Answer: (c) $3^{x/12} = 0.1 \Leftrightarrow \log 3^{x/12} = \log 0.1 \Leftrightarrow \frac{1}{12}x\log 3 = -1 \Leftrightarrow x = -12/(\log 3) \approx -25.1508$

188. Find the solution of the equation $\left(\frac{1}{4}\right)^x = 81$ correct to four decimal places.

(a) -3.1699 (b) -3.0501 (c) -2.7593 (d) -2.1175 (e) -1.7659

Answer: (a) $\left(\frac{1}{4}\right)^x = 81 \Leftrightarrow x\log\frac{1}{4} = \log 81 \Leftrightarrow x = -\frac{\log 81}{\log 4} \approx -3.1699$

189. Find the solution of the equation $10^{1-x} = 4^x$ correct to four decimal places.

(a) -0.0365 (b) 0.0265 (c) 0.3785 (d) 0.3932 (e) 0.6242

Answer: (e) $10^{1-x} = 4^x \Leftrightarrow 1 - x = x\log 4 \Leftrightarrow x = \frac{1}{1+\log 4} \approx 0.6242$

190. Find the solution of the equation $e^{2-5x} = 8$ correct to four decimal places.

(a) -0.1875 (b) -0.0159 (c) 0.2056 (d) 0.3869 (e) 1.6532

Answer: (b) $e^{2-5x} = 8 \Leftrightarrow \ln e^{2-5x} = \ln 8 \Leftrightarrow 2 - 5x = \ln 8 \Leftrightarrow x = \frac{1}{5}(2 - \ln 8) \approx -0.0159$

191. Solve the equation $e^x = 10$ for x.

(a) $x = -\frac{1}{10}$ (b) $x = 2\ln 2$ (c) $x = \ln 5$ (d) $x = \ln 10$ (e) $x = 2\ln 5$

Answer: (d) $e^x = 10 \Leftrightarrow x = \ln 10$

192. Solve the equation $e^{1-4x} = 2$ for x.

(a) $x = \frac{1}{2}(\ln 2 - 1)$ (b) $x = 2\ln 2 - 1$ (c) $x = \frac{3}{4}\ln\frac{1}{2}$
(d) $x = \frac{2}{3}(2 - \ln 3)$ (e) $x = \frac{1}{4}(1 - \ln 2)$

Answer: (e) $e^{1-4x} = 2 \Leftrightarrow \ln e^{1-4x} = \ln 2 \Leftrightarrow 1 - 4x = \ln 2 \Leftrightarrow x = \frac{1}{4}(1 - \ln 2)$

193. Solve the equation $2 \log x = \log 2 + \log(x + 4)$ for x.

Answer: $2 \log x = \log 2 + \log(x + 4) \Leftrightarrow \log x^2 = \log(2(x + 4)) \Leftrightarrow x^2 = 2x + 8 \Leftrightarrow x^2 - 2x - 8 = 0 \Leftrightarrow (x + 4)(x + 2) = 0 \Leftrightarrow x = 4, -2$. But –2 is not a solution because negative numbers do not have logarithms. So x = 4 is the only solution.

194. Solve the equation $\log_4 x + \log_4(x + 1) = \log_4 30$.

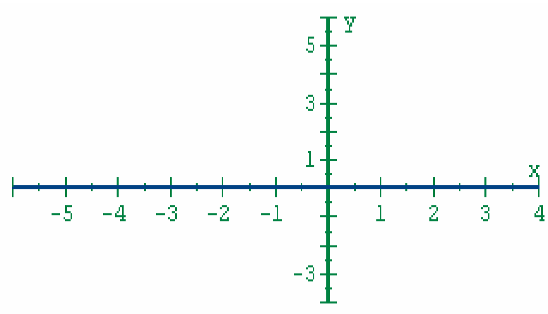
Answer:
$\log_4 x + \log_4(x + 1) = \log_4 30 = \log_4 x(x + 1) = \log_4 30 \Leftrightarrow x(x + 1) = 30 \Leftrightarrow x^2 + x - 30 = 0 \Leftrightarrow (x + 6)(x - 5) = 0 \Leftrightarrow$ x = -6 or 5. But x = -6 is inadmissible, so x = 5 is the only solution.

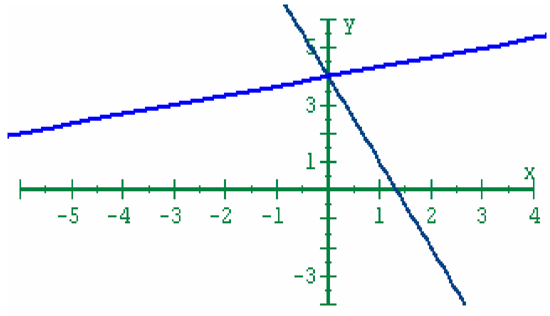
195. Solve the equation $\log_{16} x + \log_{15}(x - 2) = 1$ for x.

Answer: $\log_{16} x + \log_{15}(x - 2) = 1 \Leftrightarrow \log_{15} x(x - 2) = 1 \Leftrightarrow x(x - 2) = 15^1 \Leftrightarrow x^2 - 2x - 15 = 0 \Leftrightarrow (x - 5)(x + 3) = 0 \Leftrightarrow$ x = 5 or -3. But x = -3 is inadmissible, so the only solution is x = 5.

196. Graph the pair of lines $\begin{cases} 3x + y = 4 \\ -x + 3y = 12 \end{cases}$ on a single set of axes. Determine whether the lines are parallel or not, and if they are not parallel, estimate the coordinates of their point of intersection from your graph.

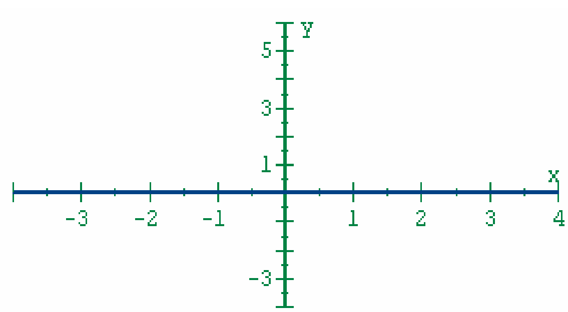
Answer: $\begin{cases} 3x + y = 4 \\ -x + 3y = 12 \end{cases}$ Not parallel. Intersect at (0, 4).

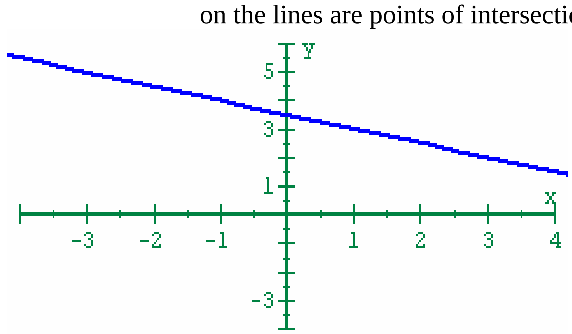




197. Graph the pair of lines $\begin{cases} 2x + 4y = 14 \\ x + 2y = 7 \end{cases}$ on a single set of axes. Determine whether the lines are parallel or not, and if they are not parallel, estimate the coordinates of their point of intersection from your graph.

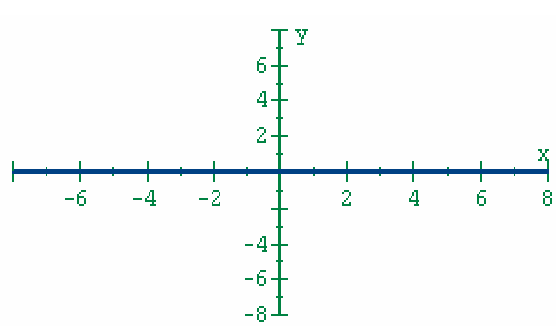
Answer: $\begin{cases} 2x + 4y = 14 \\ x + 2y = 7 \end{cases}$ The lines are identical. All points on the lines are points of intersection.

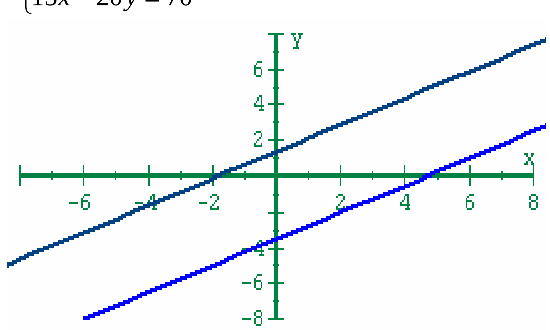




198. Graph the pair of lines $\begin{cases} -6x + 8y = 11 \\ 15x - 20y = 70 \end{cases}$ on a single set of axes. Determine whether the lines are parallel or not, and if they are not parallel, estimate the coordinates of their point of intersection from your graph.

Answer: $\begin{cases} -6x + 8y = 11 \\ 15x - 20y = 70 \end{cases}$ Parallel. No intersection





199. Solve the system $\begin{cases} 2x - y = 6 \\ 9x - 2y = -4 \end{cases}$ using the substitution method.

(a) (-6, -5) (b) $\left(-\frac{16}{5}, -\frac{62}{5}\right)$ (c) $\left(-\frac{13}{5}, -\frac{52}{5}\right)$ (d) $\left(-\frac{17}{10}, -\frac{105}{10}\right)$ (e) (-1, 1)

Answer: (b)
2x – y = 6 ⇔ y = 2x – 6. Substituting for y into 9x – 2y = -4 gives 9x – 2(2x – 6) = -4 ⇔ 5x = -16 ⇔
x = $-\frac{16}{5}$, and so $y = 2(-\frac{16}{5}) - 6 = -\frac{62}{5}$. Thus the solution is $\left(-\frac{16}{5}, -\frac{62}{5}\right)$.

200. Solve the system $\begin{cases} 5x + 3y = 12 \\ 4x - 2y = 14 \end{cases}$ using the elimination method. If the system has infinitely many solutions, write the general form for all the solutions.

(a) (3, -4) (b) (2, 2) (c) (1, -2) (d) (2, -3) (e) (3, -1)

Answer: (e)
$\begin{cases} 5x + 3y = 12 \\ 4x - 2y = 14 \end{cases} \Leftrightarrow \begin{cases} 10x + 6y = 24 \\ 12x - 6y = 42 \end{cases}$. Adding gives 22x = 66 ⇔ x = 3, and so 4(3) – 2y = 14 ⇔ y = -1.
So the solution is (3, -1).

201. Write a system of equations that corresponds to the matrix $\begin{bmatrix} 2 & 3 & 5 & 6 \\ -1 & -1 & 5 & 2 \\ -2 & 3 & 0 & 11 \end{bmatrix}$.

Answer:

$$\begin{bmatrix} 2 & 3 & 5 & 6 \\ -1 & -1 & 5 & 2 \\ -2 & 3 & 0 & 11 \end{bmatrix} \Leftrightarrow \begin{cases} 2x + 3y + 5z = 6 \\ -x - y + 5z = 2 \\ -2x + 3y = 11 \end{cases}$$

202. Write a system of equations that corresponds to the matrix $\begin{bmatrix} 5 & 2 & 1 & 4 & 0 \\ -3 & 1 & 1 & 0 & 5 \\ 2 & 3 & 4 & 0 & 1 \\ 0 & 1 & 1 & 0 & -3 \end{bmatrix}$.

Answer:

$$\begin{bmatrix} 5 & 2 & 1 & 4 & 0 \\ -3 & 1 & 1 & 0 & 5 \\ 0 & 1 & 1 & 0 & -3 \end{bmatrix} \Leftrightarrow \begin{cases} 5x + 2y + z + 4w = 0 \\ -3x + y + z = 5 \\ 2x + 3y + 4z = 1 \\ y + z = -3 \end{cases}$$

203. Use Gaussian elimination to solve the system $\begin{cases} 3x - y = -9 \\ -x + 2y + 3z = 17 \\ x + y + z = 4 \end{cases}$.

(a) (1, 2, -3) (b) (-2, 3, 3) (c) (4, 2, -2) (d) 5, 1, 6) (e) (-2, -2, 7)

Answer: (b)

$$\begin{bmatrix} 3 & -1 & 0 & -9 \\ -1 & 2 & 3 & 17 \\ 1 & 1 & 1 & 4 \end{bmatrix} R_1 \leftrightarrow R_3 \begin{bmatrix} 1 & 1 & 1 & 4 \\ -1 & 2 & 3 & 17 \\ 3 & -1 & 0 & -9 \end{bmatrix} \begin{matrix} R_2 + R_1 \to R_2 \\ R_3 - 3R_1 \to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & -4 & -3 & -21 \end{bmatrix}$$

$3R_3 + 4R_2 \to R_3$ $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 7 & 21 \end{bmatrix}$ Then 7z = 21 $\Leftrightarrow$ z = 3; 3y + 4(3) = 21 $\Leftrightarrow$ y = 3; x + 3 + 3 = 4 $\Leftrightarrow$ x = -2.

Therefore, the solution is (-2, 3, 3).

204. Use Gaussian elimination to solve the system $\begin{cases} 2x - 3y + 5z = 15 \\ 4x + 2y - 3z = -6 \\ 6x + y + z = 9 \end{cases}$.

(a) $\left(\frac{1}{3}, -\frac{2}{3}, 4\right)$ (b) (-2, 6, -1) (1, 0, -3) (d) $\left(\frac{1}{2}, 2, 4\right)$ (e) (8, 2, 3)

Answer: (d)

$$\begin{bmatrix} 2 & -3 & 5 & 15 \\ 4 & 2 & -3 & -6 \\ 6 & 1 & 1 & 9 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \to R_2 \\ R_3 - 3R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & -3 & 5 & 15 \\ 0 & 8 & -13 & -36 \\ 0 & 10 & -14 & -36 \end{bmatrix} 5R_2 - 4R_3 \to R_3 \begin{bmatrix} 2 & -3 & 5 & 15 \\ 0 & 8 & -13 & -36 \\ 0 & 0 & -9 & -36 \end{bmatrix}$$

-9z = -36	8y - 13z = -36	2x - 3y + 5z = 15
z = 4	8y – 52 = -36	2x – 6 + 20 = 15
	8y = 16	2x + 14 = 15
	y = 2	2x = 1
		x = ½

205. Use Gaussian elimination to solve the system $\begin{cases} 2x + 10y - 20z = 80 \\ 4x + 20y + 30z = -50 \\ -2x + 6y - 20z = 40 \end{cases}$.

(a) (2, 1, 3) (b) (10, 0, -3) (c) (15, 7, -2) (d) (14, 15, 5) (e) (-2, -6, 3)

Answer: (b)

$$\begin{bmatrix} 2 & 10 & -20 & 80 \\ 4 & 20 & 30 & -50 \\ -2 & 6 & -20 & 40 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \to R_2 \\ R_1 + R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & 10 & -20 & 80 \\ 0 & 0 & 70 & -210 \\ 0 & 16 & -40 & 120 \end{bmatrix} R_2 \leftrightarrow R_3 \begin{bmatrix} 2 & 10 & -20 & 80 \\ 0 & 16 & -40 & -210 \\ 0 & 0 & 70 & 210 \end{bmatrix}$$

206. Find all solutions (x, y) of the system of equations $\begin{cases} x^2 + y = 9 \\ x - y + 3 = 0 \end{cases}$.

(a) (-2, 1), (1, 3) (b) (-3, 0), (2, 5) (c) (1, -1), (-2, 2)
(d) (0, 1), (1, 0) (e) No solutions

Answer: (b)

$\begin{cases} x^2 + y = 9 \\ x - y + 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x^2 = 9 - y \\ y = x + 3 \end{cases} \Rightarrow x^2 = 9 - (x+3) \Leftrightarrow x^2 + x - 6 = 0 \Leftrightarrow (x + 3)(x - 2) = 0 \Leftrightarrow$
x = -3 or x = 2. The solutions are (-3, 0) and (2, 5).

207. Find all solutions (x, y) of the system of equations $\begin{cases} y = 9 - x^2 \\ y = x^2 - 9 \end{cases}$.

(a) (3, -3), (-3, 3) (b) $(\pm 3, \pm 3)$ (c) (1, 3)(-3, 1)
(d) (3, 0)(-3, 0) (e) No solution

Answer: (d)

$\begin{cases} y = 9 - x^2 \\ y = x^2 - 9 \end{cases} \Rightarrow 9 - x^2 = x^2 - 9 \Leftrightarrow 2x^2 = 18 \Leftrightarrow x = \pm 3$. Therefore, the solutions are (3, 0) and (-3, 0).

208. Find all solutions (x, y) of the system of equations $\begin{cases} x^2 + 4y^2 = \frac{5}{2} \\ 2x^2 - 4y = 12 \end{cases}$.

(a) $(\sqrt{2}, -\sqrt{2})$ (b) $(\pm\sqrt{2}, -\sqrt{2})$ (c) $(1, \frac{-3 \pm 2\sqrt{2}}{2})$

(d) $(1, \frac{-3 \pm 2\sqrt{2}}{16})$ (e) No solutions

Answer: (e)

$\begin{cases} x^2 + 4y^2 = \frac{5}{2} \\ 2x^2 - 4y = 12 \end{cases} \Leftrightarrow \begin{cases} 2x^2 + 8y^2 = 5 \\ 2x^2 - 4y = 12 \end{cases}$. Subtracting the two equations gives $8y^2 + 4y = -7 \Leftrightarrow 8y^2 + 4y + 7 = 0 \Leftrightarrow y =$

$\frac{-4 \pm \sqrt{16 - 4(8)(7)}}{2(8)}$ which is not a real number. Therefore, there are no solutions.

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$, and $\pm\frac{1}{6}$